16P204 (Pages: 2) Name..........................

Reg. No.........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC 15P MT2 C09 - PDE & INTEGRAL EQUATIONS

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

*(Answer **all** questions, each question has 1 weightage)*

1. Show that if there is a functional relation between two function u(x, y) and v(x, y)not involving x and y explicitly, then $\frac{\partial(u,v)}{\partial(x,y)} = 0$.
2. Find the partial differential equation by eliminating the parameters 'a' and 'b' from the equation $z = x + ax^2y^2 + b$.
3. Determine the region D in which the two equations $xp - yq - x = 0$ and $x^2p + q - xz = 0$ are compatible.
4. Explain Cauchy problem for a non linear equation.
5. Determine the Monge cone in the case of $p^2 + q^2 = 1$ with vertex (0, 0, 0).
6. Write the classification of the equation : $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$
7. Prove that the solution of the Dirichlet problem if it exist is unique.
8. State the Cauchy problem for the equation $Au_{xx} + Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$, where A, B and C are functions of x and y. Also give an example.
9. What is the Neumann problem for the upper half plane.
10. What is Riemann function?
11. Obtain the exact solution of $y(x) = \lambda \int_0^1 x\xi\; y(\xi)\, d\xi + 1$.
12. If $y''(x) = F(x)$ and y satisfies the initial conditions $y(0) = y_0$, $y'(0) = y_0'$, show that $y(x) = \int_0^x (x - \xi)F(\xi)d\xi + y_0' x + y_0$.
13. Define separable kernel and give an example of it.
14. Determine the resolvent kernel associated with $K(x, \xi) = \cos(x + \xi)$ in $(0, 2\pi)$ in the form of power series in λ obtaining the first three terms.

(14 x 1= 14 Weightage)

Part B

*(Answer **any seven** from the following ten questions (15-24), each question has 2 weightage)*

15. Find the general integral of the equation $z(xp - yq) = y^2 - x^2$.
16. Find the complete integral of the equation $(p^2 + q^2)y - qz = 0$ by Charpit's method.

17. Solve by Jacobi's method the equation : $z^3 = pqxy$.

18. Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through $x = 1, y = 0, z = s$.

19. Reduce the equation $u_{xx} - 4x^2u_{yy} = \frac{u_x}{x}$ into canonical form.

20. Solve $y_{tt} - c^2y_{xx} = 0, \ 0 < x < 1, t > 0,$

$y(0, t) = y(l, t) = 0,$

$y(x, 0) = x(1 - x), 0 \le x \le 1,$

$y_t(x, 0) = 0, 0 \le x \le 1.$

21. Show that the solution of the Neumann problem is unique up to the addition of a constant.

22. Transform the problem $y''(x) + xy = 1; y(0) = y(1) = 0$ to a Fredholm integral equation using Green's function.

23. Solve $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$, using Neumann series.

24. If $y_m(x)$ and $y_n(x)$ are characteristic functions of the equation with a real symmetric kernel corresponding respectively to two different characteristic numbers λ_m and λ_n of homogeneous Fredholm equation $y(x) = \lambda \int_a^b K(x, \xi)\, y(\xi)\, d\xi$, then prove that $y_m(x)$ and $y_n(x)$ are orthogonal over the interval (a,b).

(7 x 2= 14 Weightage)

Part C

(Answer any two from the following four questions (25-28), each question has 4 weightage)

25. Show that the pfaffian differential equation $(1 + yz)dx + x(z - x)dy - (1 + xy)dz = 0$ is integrable and find the corresponding integral.

26. Using the method of characteristics, find an integral surface of $p^2x + qy - z = 0$ which passes through the curve $x + z = 0, y = 1$.

27. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

28. Consider the equation $y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x + \xi)\, y(\xi)\, d\xi$.

(a) Determine the characteristic values of λ and the corresponding characteristic functions.

(b) Express the solution in the form $y(x) = F(x) + \lambda \int_0^{2\pi} \Gamma(x, \xi; \lambda)\, F(\xi)\, d\xi$ when λ is not characteristic.

(2 x 4= 8 Weightage)

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