16P255 (Pages: 2) Name…………………

Reg. No……………….

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST2 C08 - PROBABILITY THEORY

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A Answer *all* questions

1. Define Convergence in probability of sequence of random variables.
2. Find the distribution function of a random variable with density $f(x) = \frac{1}{b-a}, a < x < b, 0$ otherwise.
3. Define mathematical expectation of real valued Borel function.
4. Check Convergence almost sure of sequence of random variables, $P\left[X_n = \mp\frac{1}{n}\right] = 1/2$
5. Show that $X_n \overset{p}{\to} X$, implies $X_n \overset{L}{\to} X$, n=1,2,…
6. State Chebyshev's inequality for independent random variables.
7. State Borell Cantelli Lemma.
8. State correspondence theorem.
9. Check whether $\phi(t) = \frac{1}{2}(1 + e^{3it})$ is a characteristic function.
10. If ϕ is a characteristic function. Is $|\phi|$ a characteristic function.
11. State Continuity Theorem.
12. State Radon-Nikodyn theorem. Give one application.

(12 x 1=12 weightage)

Part B Answer any *eight* questions

13. State and prove Jordan-decomposition theorem.
14. Let $\{X_n\}$ be sequence of iid random variables. Then $X_n \overset{a.s.}{\to} 0$ if and only if $\sum_{n=1}^{\infty} P\{|X_n| > c\} < \infty$ for all $\epsilon > 0$.
15. State and prove Kolmogorov's WLLN's.
16. Examine the convergence of

$$F_n(x) = 0, \quad \text{if } x < -n$$

$$= \frac{1}{2} + c_n \tan^{-1}(cx), \quad -n \leq x < n$$

$$= 1, \quad \text{if } x \geq n.$$

17. If $X_n \overset{a.s.}{\to} X$, show that $X_n \overset{P}{\to} X$.

18. State and prove Borel 0-1 Law.
19. Show that $\{X_n\}$ convergence in probability to a random variable X if and only if it is Cauchy in probability.

20. Show that Borel functions of independent random variables are independent.
21. Let $\{ X_n, Y_n \}$ be a sequence of pairs of random variables with $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{P} c$. Show that $X_n + Y_n \xrightarrow{L} X + c$ and $X_n Y_n \xrightarrow{L} cX$.
22. State and prove Lindeberg-Levy Central Limit theorem.
23. Prove that characteristic function is uniformly continuous on R.
24. Define conditional expectation and its properties.

(8 x 2 =16 Weightage)

Part C Answer any *two* questions.

25. (a) Show that $\{X_n\}$ converges in probability to a random variable if it is converges in r^{th} mean
    (b) Show that $\{X_n\}$ Cauchy in a.s. implies $\{X_n\}$ Cauchy in probability.
26. (a)State and prove Helly-Bray theorem
    (b) Prove $\varphi_x(t) = e^{-t^4}$, *and* $\varphi x(t) = (1 + t^4)^{-1}$, are not characteristic functions
27. State and prove Inversion theorem.
28. (a) State and prove Jordan decomposition theorem.
    (b) State and prove the necessary and sufficient conditions for a function to be a distribution function.

(2 x 4 = 8 Weightage)

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