15P304 (Pages:2) Name…………………

Reg. No…………….

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCSS - PG)

(Mathematics)

CC15P MT3 C14 - LINEAR PROGRAMMING AND ITS APPLICATIONS

(2015 Admission)

Time : Three Hours Maximum : 36 Weightage

PART A

Answer **All** Questions. Each question has weightage 1

1. Define polytope and give an example for polytope with one vertex.
2. Prove that all internal points of a convex set K themselves constitute a convex set.
3. Define Convex hull with example..
4. Find $H(x)$ for $f(x) = x_1^3 + 2x_2^3 + 3x_1x_2x_3 + x_3^2$
5. Find the directional derivative of $f(x) = 6x_2^2 - 18x_2x_3 - 6x_3x_1 + 2x_1x_2 - 7x_1 + 5x_2 - 6x_3 - 4$ at X_0 in the direction of the vector $Y = [1 \quad 1 \quad 1]'$
6. Define convex functions.
7. Define feasible solutions and basic feasible solutions
8. Prove that dual of the dual is primal.
9. Define Transportation matrix with example.
10. Define loop in transportation array.
11. Define triangular basis.
12. Differentiate between ILP and MILP.
13. Define pure strategy and mixed strategy.
14. Find the optimal strategy and value of the game $\begin{bmatrix} 2 & -1 & -2 \\ 1 & 0 & 1 \\ -2 & -1 & 2 \end{bmatrix}$.

(14×1=14 Weightage)

PART B

Answer any **Seven** Questions. Each question has weightage 2.

15. Prove that the convex polyhedron is a convex set.
16. Let $f(X,Y)$ be such that both $\max\min f(X,Y)$ and $\min\max f(X,Y)$ exist. Then prove that the necessary and sufficient condition for the existence of saddle point (X_0, Y_0) of $f(X,Y)$ is that $f(X_0,Y_0) = \max\min f(X,Y) = \min\max f(X,Y)$
17. Find the relative maxima and minima and saddle points, if any, of $f(x) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 25$.
18. Examine $f(x) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 16x_2x_3$ for relative extrema.
19. Prove that a vertex of S_F is a basic feasible solution.

20. Prove that the optimum value of $f(x)$ of the primal, if it exists, is equal to the optimum value of $\varphi(Y)$ of the dual.
21. Explain Caterer problem.
22. Describe a rectangular game as an LP problem
23. Solve the game with the given payoff matrices $\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$.
24. Given $x_{13} = 50$ units, $x_{14} = 20$ units, $x_{21} = 55$ units, $x_{31} = 30$ units, $x_{32} = 35$ units and $x_{34} = 25$ units. Is it an optimal solution to the transportation problem.

					Available Units:
	6	1	9	3	70
	11	5	2	8	55
	10	12	4	7	90
Required units:	85	35	50	45	

If not, modify it to obtain a better feasible solution

(7×2=14 Weightage)

PART C

Answer any **Two** Questions. Each question has weightage 4.

25. Solve graphically the game whose pay off matrix is $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$.
26. Explain cutting plane method. And solve the problem
    $maximize\ 3x_1 + 4x_2\ \ Subject\ to\ \ 4x_1 + 3x_2 \geq 12,\ \ x_1 + 2x_2 \leq 2\ ,\ x_1, x_2 \geq 0.$
27. Prove that if a set K is non empty, closed, bounded and convex, then *(i)* K has at least one vertex *(ii)* every point of K is a convex linear combination of its vertices.
28. Find the maximum and minimum values of $|X|^2, X \epsilon E_3$ subject to the constraints

$$g_1(X) = \frac{x_1^2}{4} + \frac{x_2^2}{5} + \frac{x_3^2}{25} - 1 = 0, \qquad g_2(X) = x_1 + x_2 - x_3 = 0$$

(2×4=8 Weightage)

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