**15P303** (Pages:2) Name……………………

Reg. No………………

**THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016**

(CUCSS - PG)

(Mathematics)

# CC15P MT3 C13 - TOPOLOGY II

(2015 Admission)

Time : Three Hours Maximum : 36 Weightage

## Part A

Answer **all** questions.

Each question carries 1 weightage.

1. Prove that the intersection of any family of boxes is a box.
2. Define a cube and a Hilbert cube.
3. Let $\mathcal{S}$ be a sub base for a topological space X. Then show that X is completely regular if and only if for each V$\in\mathcal{S}$ and for each x$\in$V, there exists a continuous function $f: X \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \notin V$.
4. Show that a topological product of spaces is Tychonoff if and only if each coordinate space is so.
5. Let $\{Y_i: i\epsilon I\}$ be a family of sets, X be any set and for each $i\epsilon I$, define $f_i: X \to Y_i$. Show that the evaluation function is the only function from X into $\Pi Y_i$ whose composition with the projection $\pi_i: \Pi Y_i \to Y_i$ equals $f_i$ for all $i\epsilon I$.
6. Show that the evaluation function of a family of functions is one-to-one if and only if that family distinguishes points.
7. Define homotopy.
8. Let X be a space; and $x_o$ be a point of X. Define the fundamental group of X relative to the base point $x_o$.
9. Define a covering space.
10. Show that the continuous image of a countably compact space is countably compact.
11. Define sequential compactness.
12. Prove that every locally compact, Hausdorff space is regular.
13. Describe the one-point compactification of a topological space X.
14. Prove that a finite union of totally bounded set is totally bounded.

**(14 x1=14 weightage)**

## Part B

Answer any **7** questions.

Each question carries 2 weightage

15. Let A be a closed subset of a normal space X and suppose $f: A \to (-1,1)$ is continuous. Show that there exists a continuous function $F: X \to (-1,1)$ such that $F(x) = f(x)$ for all $x \epsilon A$.
16. Let $X = \Pi X_i$, each $X_i$ being a topological space. Suppose $\{x_n\}$ is a sequence in X and that $x \epsilon X$. Prove that $\{x_n\}$ converges to $x$ in X if and only if for each $i \epsilon I$, the sequence $\{\pi_i(x_n)\}$ converges to $\pi_i(x)$ in $X_i$.
17. Define productive property. Show that $T_2$ is a productive property.
18. Prove that a product of topological spaces is path connected if and only if each coordinate space is path connected.
19. Prove that a topological space is completely regular if and only if the family of all continuous real-valued functions on it distinguishes points from closed sets.
20. Show that the evaluation function of a family of functions which distinguishes points from closed sets is open.
21. Show that the relation $\simeq_p$ (path homotopy) is an equivalence relation.
22. Let X be a $T_1$ space. Prove that every infinite subset of X has an accumulation point if and only if every sequence in X has a cluster point.
23. Prove that a subspace of a locally compact, Hausdorff space is locally compact if and only if it is open in its closure.
24. Prove that every compact metric space is complete.

**(7 x2=14 weightage)**

## Part C

Answer any **two** questions.

Each question carries 4 weightage

25. Prove that metrisability is a countably productive property.
26. Prove that a second countable space is metrisable if and only if it is $T_3$.
27. State and prove Alexander Sub-base theorem.
28. Prove that the fundamental group of the circle is infinite cyclic.

**(2 x4=8 weightage)**

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