18P308 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Regular/Supplementary/Improvement)

(CUCSS-PG)

(Mathematics)

CC17P MT3 C15/CC18P MT3 C15 - PDE AND INTEGRAL EQUATIONS

(2017 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Give two parametric representation of a surface $x^2 + y^2 + z^2 = a^2$
2. Show that if there is a functional relation between two function $u(x, y)$ and $v(x, y)$ not involving x and y explicitly, then $\frac{\partial(u,v)}{\partial(x,y)} = 0$
3. Determine the region D in which the two equations $p^2 + q^2 - 1 = 0$ and $(p^2 + q^2)x - pz = 0$ are compatible.
4. Find the complete integral of $z - px - qy - p^2 - q^2 = 0$
5. Determine the Monge cone in the case of $p^2 + q^2 = 1$ with vertex $(0, 0, 0)$
6. Write the classification of the equation: $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$
7. Prove that the solution of the Dirichlet problem if it exist is unique.
8. State Harnack's theorem.
9. Define the domain of dependence and range of influence in the case of a one dimensional wave equation.
10. State Riemann function.
11. Obtain the exact solution of $y(x) = \lambda \int_0^1 x\xi \, y(\xi)\, d\xi + 1$
12. If $y''(x) = F(x)$ and y satisfies the initial conditions $y(0) = y_0$, $y'(0) = y_0'$, show that $y(x) = \int_0^x (x - \xi)F(\xi)d\xi + y_0' x + y_0$
13. Explain the condition for the existence of reduntant solution of an integral equation.
14. Determine the resolvent kernel associated with $K(x, \xi) = x\xi$ in $(0, 1)$ in the form of power series in λ obtaining the first three terms.

(14 x 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Find the general integral of the equation $(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z)$
16. Find the integral corresponding to $yzdx + (x^2y - xz)dy + (x^2z - xy)dz = 0$

17. Solve by Jacobi's method the equation $z^3 = pqxy$
18. Find the integral surface of the equation $x^3p + y(3x^2 + y)q = z(2x^2 + y)$ which passes through $x = 1, y = s, z = s(1 + s)$
19. Find the solution of $(n-1)^2 u_{xx} - y^{2n}u_{yy} = ny^{2n-1}u_y$, where n is a positive integer.
20. Solve $y_{tt} - c^2y_{xx} = 0, 0 < x < 1, t > 0,$

    $y(0,t) = y(l, t) = 0,$

    $y(x, 0) = x(1 - x), 0 \leq x \leq 1,$

    $Y_t(x, 0) = 0, 0 \leq x \leq 1$
21. Prove that the solution $u(x,t)$ of the differential equation $u_t - ku_{xx} = F(x,t), 0 < x < l$, $t > 0$ satisfying the initial conditions $u(x, 0) = f(x), 0 \leq x \leq l$ and the boundary conditions $u(0, t) = u(l, t) = 0, t \geq 0$ is unique.
22. Transform the problem $\frac{d^2y}{dx^2} + y = x, y(0) = 1, y(1) = 0$ to a Fredholm integral equation.
23. Solve the Fredholm integral equation by iterative method: $y(x) = 1 + \lambda \int_0^1 (1 - 3x\,\xi)\, y(\xi)\, d\xi$
24. Determine the iterated kernel $K_2(x, \xi)$ associated with $K(x, \xi) = |x - \xi|$ in $(0, 1)$

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. State and prove the necessary and sufficient condition that the PDE $\overline{X}.d\overline{r} = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$ be integrable is that $\overline{X}.Curl\,\overline{X} = 0$
26. Using the method of characteristics, find an integral surface of $pq = xy$ which passes through the line $x = z, y = 0$
27. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.
28. Consider the equation $y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x + \xi)\, y(\xi)\, d\xi$

    (a) Determine the characteristic values of λ and the corresponding characteristic functions.

    (b) When $F(x) = 1$, what is the solution of the above integral equation?

(2 x 4 = 8 Weightage)

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