3 copy

15P402 (Pages: 2) Name......................

Reg. No......................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCSS - PG)

(Mathematics)

CC15P MT4 C16 – DIFFERENTIAL GEOMETRY

(2015 Admission)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer *all* Questions

Each Question Carries *1* weightage

1. Define the terms level sets at height C with an example.
2. Define parameterized curve in R^{n+1}. What is the corresponding velocity vector?
3. Sketch the following vector fields in R^2.
   (i) $X(p) = (0,2)$
   (ii) $X(p) = p$
4. Define divergence of smooth vector field X on $U \subseteq R^2$.
5. Define n-surface in R^{n+1}.
6. Let $f(x_1, x_2, \dots, x_{n+1}) = x_1.x_2.\dots, x_{n+1} + 1$. Show that $f^{-1}(c)$ is an n-surface when $c \neq 1$.
7. Let $\overline{X}$ and $\overline{Y}$ be two smooth vector fields. Then $(\dot{\overline{X} + \overline{Y}}) = \dot{\overline{X}} + \dot{\overline{Y}}$.
8. Define covariant derivative.
9. Prove that the function which sends $\overline{v}$ into $\nabla_{\overline{v}}f$ is a linear map.
10. Define geodesic in an n-surface.
11. Write Frenet formula for a plane curve.
12. Mention the first fundamental for of S at the point p.
13. Write the inverse function theorem for an n-surface.
14. If $\overline{X}$ is parallel along α, then $\overline{X}$ has constant length.

(14x1= 14 weightage)

Part B

Answer any *seven* Questions

Each question carries *2* weightage

15. Sketch the level sets $f^{-1}(c)$ for n = 0,1 for the function $f(x_1, x_2, \dots, x_{n+1}) = x_{n+1}$ for c = 0,1,2.
16. The gradient of 'f' at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.

17. Let $S \subseteq R^{n+1}$ be a connected n - surface in R^{n+1}. Then there exist on S exactly two smooth unit normal vector fields.

18. For each a, b, c, d ∈ R, the parameterised curve $\alpha(t) = (\text{Cos}(at + b), \text{Sin}(at + b), ct + d)$ is a geodesic in the cylinder $x_1^2 + x_2^2 = 1$ in R^3.

19. Let $\overline{X}$ and $\overline{Y}$ be two smooth vector fields tangent to S along α then show that $(\overline{X}.\overline{Y})' = \overline{X}'.\overline{Y} + \overline{X}.\overline{Y}'$

20. Let S be an n-surface in R^{n+1}, let p, q ∈ S and let α be a piecewise smooth parameterised curve from p to q. Then parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.

21. The normal component of the acceleration is same for all parameterised curves α in S passing through p with velocity $\overline{v}$.

22. Show that if C is a connected oriented plane curve and $\tilde{C}$ is the same curve with the opposite orientation, then length (C) = length ($\tilde{C}$).

23. On each compact oriented n-surface S in R^{n+1}, there exist a point p such that the second fundamental form at p is definite.

24. Find the Gaussian curvature of $\varphi(t, \theta) = (\text{Cos}\theta, \text{Sin}\theta, t)$.

(7x2= 14 weightage)

Part C

Answer any *two* Questions

Each question carries **4** weightage

25. Let U be an open set in R^{n+1} and let f: U → R be smooth. Let $p \in U$ be a regular point of f and let c = f(p). Then the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$.

26. State and prove inverse function theorem for n-surface.

27. Obtain Gaussian curvature of an ellipsoid *S*.

28. Prove that the Weingarten map L_p is self adjoint.

(2x4= 8 weightage)

****