16P446 (Pages:2) Name………………….

Reg. No………………..

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MARCH 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15 PST4 C13 –MULTIVARIATE ANALYSIS

(Statistics)

(2015 Admission onwards)

Time: 3 Hours Maximum:36 Weightage

Part A

Answer ***all*** questions. Each question carries ***1*** weightage.

1. Let $\mathbf{Y} \sim N_p(\mathbf{0}, \boldsymbol{I})$ and let **A** and **B** be p × p symmetric idempotent matrices. (i) Identify the distributions of $\mathbf{Y}^T\mathbf{AY}$ and $\mathbf{Y}^T\mathbf{BY}$ and (ii) write down the condition for $\mathbf{Y}^T\mathbf{AY}$ and $\mathbf{Y}^T\mathbf{BY}$ to be independent.
2. Let $\mathbf{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma} = \sigma^2\boldsymbol{I}$. Compute $E([\mathbf{X}-\boldsymbol{\mu}]^T \boldsymbol{\Sigma}^{-1} [\mathbf{X}-\boldsymbol{\mu}])$.
3. Define (i) partial correlation coefficient and (ii) partial regression coefficient.
4. Define Multiple Correlation Coefficient
5. What is meant by generalized variance?
6. State and prove additive property of Wishart's distribution.
7. Distinguish between multiple correlation and canonical correlation
8. Show that Hotelling's T^2 statistic is invariant under transformation.
9. Establish the relation between Mahalanobis D^2 statistic and Hotelling's T^2 statistic.
10. What is Fisher's discriminant function? Discuss its uses.
11. What is the importance of principal component analysis?
12. Explain an orthogonal factor model.

(12×1=12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries ***2*** weightage.

13. If $X = [X^{(1)}, X^{(2)}]' \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ .Derive the conditional distribution of $X^{(1)}$ given $X^{(2)}$.
14. If $\mathbf{X} \sim N_3\left(\begin{pmatrix}1\\2\\4\end{pmatrix}, \begin{bmatrix}4 & 3 & 1\\2 & 5 & 3\\1 & 2 & 3\end{bmatrix}\right)$. Find the conditional distribution of $X_1/(X_2 = -1, X_3 = 4)$.
15. If $\boldsymbol{X} \sim \boldsymbol{N}(\boldsymbol{\mu}, \Sigma)$ and Σ is non-singular then derive the distribution of $(\boldsymbol{X}-\boldsymbol{\mu})' \Sigma^{-1} (\boldsymbol{X}-\boldsymbol{\mu})$.
16. Obtain the null distribution of sample correlation.
17. How would you estimate the canonical correlation and canonical variables?
18. Obtain the characteristic function of a Wishart's distribution.

19. Derive the sampling distribution of Hotelling’s T^2 statistic.
20. Explain the testing procedure to test the independence of sub vectors of a multivariate normal distribution.
21. Obtain the characteristic function of a multivariate normal distribution.
22. What is Fisher - Behran’s problem? Explain.
23. Explain the classification problem with a suitable example.
24. Describe factor analysis and its significance in data analysis.

(8×2=16 Weightage)

Part C

Answer any ***two*** questions. Each question carries ***4*** weightage.

25. Derive the density function of a multivariate normal distribution.
26. Derive the distribution of sample correlation coefficient when the population correlation coefficient is zero.
27. Establish the relation between principal component & eigen structure of the variance correlation matrices.
28. Derive the likelihood ratio test for testing the equality of population mean vectors of two independent samples drawn from two multivariate Normal distributions when their population covariance matrices are equal.

(2×4=8 Weightage)

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