D 33337 (Pages : 3) Name..................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS

(2010 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions, each carry* 1 *weight.*

I. Define curl of a vector field. Explain the circulation of a fluid around a differential loop in the xy-plane in terms of curl.

II. Resolve the circular cylindrical unit vectors into their Cartesian components.

III. Define "Hermitian matrices" and "Unitary matrices". Give example to each case.

IV. What do you mean by tensors ?

V. Explain the regular and irregular singularities of Bessel's equation

$$x^2 y'' + xy' + (x^2 - n^2) y = 0.$$

VI. Define Wronskian of functions. Explain the idea of linear dependence and independence of functions in tems of Wronskian.

VII. State and prove Bessel's Inequality.

VIII. Define Euler infinite limit definition of gamma function. Deduce that $\Gamma(z+1) = z\Gamma(z)$.

IX. Define spherical Bessel functions.

X. Prove that $H_{2n+1}(0) = 0$.

XI. State Fourier series formula for a periodic function of period 2L in the interval (– L, L).

XII. Explain the conditions for the validity of Fourier cosine transform formula.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions, each carry* 6 *weights.*

XIII. (a) Write $x^2 + 2xy + 2yz + z^2$ as a sum of squares form in a rotated co-ordinate system.

(b) Expressing cross products in terms of Levi-Civita symbols, derive the

$$\textbf{BAC} - \textbf{CAB} \text{ rule : } \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \times \mathbf{C}) - \mathbf{C}(\mathbf{A} \times \mathbf{B}).$$

Turn over

XIV. Explain Frobenius method for the series solution of ordinary differential equations. Illustrat the linear oscillator equation

$$\frac{d^2 y}{dx^2} + w^2 \; y = 0.$$

XV. (a) Obtain Rodrigues's formula for Legendre polynomials. Deduce first three Legendre polyno

(b) Prove that

$$\frac{\sin x}{x} = \int_0^{\pi/2} J_0 \,(x \cos \theta) \cos \theta \; d\,\theta.$$

XVI. (a) Find the Fourier series representation of

$$f\,(x) = \begin{cases} -x, & -\pi < x \le 0 \\ x, & 0 \le x < \pi. \end{cases}$$

Also deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.

(b) Using partial fraction expansion, find inverse Laplace transform of $\frac{s}{(s+a)\,(s^2+b^2)}$

(2 × 6 = 12 wei

Section C

Answer any **four** *questions, each carry* 3 *weights.*

XVII. Explain the physical significance of the divergence.

XVIII. Halley's comet has a period of about 76 years and its closet distance from the sun is 9 × What is its greatest distance from the sun ?

XIX. Find the eigen values and corresponding orthonormal eigen vectors of

$$A = \begin{pmatrix} 5 & 0 & \sqrt{3} \\ 0 & 3 & 0 \\ \sqrt{3} & 0 & 3 \end{pmatrix}.$$

XX. Using Gram-Schmidt Orthogonalization process, form an orthonormal set from the set of functions $u_n(x) = x^n$, $n = 0, 1, 2, \ldots$ in the interval $0 \le x < \infty$ with the density function as $w(x) = e^{-x}$.

XXI. Prove that

$$\Gamma(1+z)\,\Gamma\left(z+\frac{1}{2}\right) = 2^{-2z}\sqrt{\pi}\,\Gamma(2z+1).$$

Deduce that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

XXII. Find the Fourier transform of :

$$f(x) = \begin{cases} 1, & |x| < 1, \\ 0, & |x| > 1. \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

(4 × 3 = 12 weightage)