D 72916 (Pages : 3) Name..................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Statistics

ST IC 05—DISTRIBUTION THEORY

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*

Each question carries a weightage of 1.

1. When will you say that a property is characteristic to a probability distribution. Discuss on a characterization result for any one distribution of your choice.

2. A bag contains 10 coins-one worth ten ruppees and each of the other worth five ruppees. A person draws one coin at a time without replacement until he draws the ten ruppee coin. What is the expected total amount that he has drawn (including the last coin worth ten ruppees).

3. Examine the validity of the statement "the characteristic function always exists for probability distributions". If $\phi(t)$ is the characteristic functions, write down the relationship between $\phi(t)$ and $\phi(-t)$.

4. A bivariate random vector (X, Y) is such that $P(X > x, Y > y) = \exp(-ax^{\beta} - by^{\beta} - \theta x^{\beta} y^{\beta})$
$x, y > 0 ; a, b, \beta > 0 ; \theta \geq 0.$

   Obtain the marginal distributions. For what value of θ will X and Y be independent.

5. Eventhough the normal distribution has very nice large sample properties, it is rarely used in life time studies as well as in reliability analysis. Justify the use of the exponential model in this senario.

6. Define the student's t statistic. Examine whether the student's t distribution is symmetric. Also obtain the limiting distribution of the student's t as the degrees of freedom n tends to infinity.

7. If X has an F distribution with (m_1, n_2) degrees of freedom, obtain the distribution of $1/x$. Discuss on the utility of this result.

8. Define the non-central Chi-square statistic. When will this reduce to the central Chi-square ? Also state the reproductive property of the non-central Chi-square distribution.

9. Given : $f(x, y) = 2;\ \ 0 < x < y;\ \ 0 < y < 1$. Evaluate the conditional expectation $E(Y/X = x)$.

10. If x and y are two independent random variables such that $f(x) = e^{-x}$, $x \geq$

$g(y) = 3e^{-3y}$, $y > 0$; find the distribution of $z = x/y$.

11. What do you understand by a (i) Location family; (ii) Scale family; and (iii) Location-scale Identify a distribution each which (a) belongs to (b) does not belong to the location-scale f

12. Let F (x, y) be the distribution function of (X, Y). Express $P(a < X \leq b,\ c \leq Y \leq d)$ in terms o of F (x, y) at parts a, b, c, d; $a < b$, $c < d$. Also define the moment generating function M associated with the joint distribution of X and Y.

(12 × 1 = 12 weig

Part B

Answer any **eight** *questions.*
Each question carries a weightage of 2.

13. Define probability generating function associated with a random variable. When will this to : (i) Characteristic function; and (ii) Moment generating function. If P (s) is the prob generating function associated with a non-negative integer valued random variable, sho

$$\sum_{n=0}^{\infty} s^n \, P(X \leq n) = \frac{P(s)}{1-s}.$$

14. Let X_1 and X_2 be independent random variables with distribution function $F(x) = 1 - e^{}$ some $b > 0$. If $X_{1:2}$ and $X_{2:2}$ denote the order statistics, show that $X_{2:2} - X_{1:2}$ and $X_{1:2}$ are indep

15. Obtain the characteristic function of the Cauchy distribution. Show that the arithmetic m of a sample $X_1, X_2....X_n$ of independent observations from a Cauchy distribution is also a variate.

16. If $X_1, X_2.....X_n$ is a random sample from a G (α, 1) distribution, obtain the p.d.f. of $\bar{X}$, the mean.

17. Show that the square of a non-central t statistic is a non-central F statistic. If F follow F obtain the distribution of $Z = \frac{1}{2}\log F$.

18. Let $X_1, X_2....X_n$ be a random sample of size n from the normal distribution $N(\mu, \sigma^2)$. Find t distribution of $Y = \sum_{1}^{n} a_i X_i$ and $Z = \sum_{1}^{n} b_i X_i$ where a_i and b_i are real constants. When will be independent.

19. If X is distributed as Pareto with density : $f(x) = ak^a / x^{a+1}$; $x \geq k > 0$, $a > 0$.

Show that the random variable Y with density $g(y) = y f(y) / E(X)$.

has Pareto distribution with parameters $(a - 1, k)$.

20. Suppose (Y_1, Y_2, Y_3) is multinomial (n, p_1, p_2, p_3). Show that the conditional distribution of Y_1 given that $Y_1 + Y_2 = r$ is binomial.

21. Define the lognormal distribution. Show that the distribution is skewed. Justify its utility in connection with modelling income data.

22. Let (X, Y) have the uniform distribution over the range $0 < y < x < 1$. Obtain the conditional mean and variance of X given $Y = y$.

23. Show that the normal and gamma distribution belongs to the person family of distributions.

24. What do you understand by the lack of memory property ? Identify a discrete distribution as well as a continuous distribution which possesses this property. Also show that the property holds for the above models.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. Show that :

    (i) If X follow the uniform distribution with $F(x) = x,\ 0 \le x \le 1$, then $Y = -\log X$ is unit exponential.

    (ii) If X follow the Weibull distribution with $F(x) = 1 - \exp\left(-x^{a}\right),\ x \ge 0,\ a > 0$ then $W = X^{a}$ is unit exponential.

    (iii) If X follow the Pareto law with $F(x) = 1 - x^{-a},\ a > 0,\ x \ge 1$. then $W = a \log X$ follow unit exponential.

    (iv) IF X follow the logistic distribution with $F(x) = \left(1 + e^{-ax}\right)^{-1}$, $a > 0$, x is real then $Y = \log\left(1 + e^{-ax}\right)$ is unit exponential.

26. Define the Power series family of distributions. Identify three members of the family. Obtain the m.g.f. of the distribution and deduce the mean and variance. Also obtain a recurrence relation satisfied by the cumulants.

27. $X_1, X_2 \ldots X_n$ are independent random variables with distribution function F (x). Let $X_{r:n}$ be the r^{th} order statistic and $E_{r:n} = E(X_{r:n})$. If X follow the standard exponential show that $E_{1:n} = 1/n$ for $n \ge 1$. If X follow the Uniform distribution in $0 \le x \le 1$ show that $E_{1:n} = 1/(n+1)$ $n \ge 1$ and for the Pareto model specified by $F(x) = 1 - 1/x,\ x \ge 1$ show that $E_{11} = E(X_1) = \infty$. Also show that $(n - r) E_{r:n} + r E_{r+1:n} = n E_{r:n-1}$.

28. Define the non-central Chi-square statistic and derive its distribution. When will this reduce to the Chi-square. Also discuss on the uses of the distribution.

(2 × 4 = 8 weightage)