72912 (Pages : 3) Name..

55

Reg. No.....................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Statistics

ST 1C 01—MEASURE THEORY AND INTEGRATION

(2013 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Weightage 1 for each question.

1. Examine whether $f(x) = e^{-x^2}, x \in R$ is Riemann integrable. Give the reason.
2. What do you mean by a product space ?
3. Give an example of a normed linear space.
4. Define the integral of a measurable function.
5. Define L_p space.
6. Define Riemann-Stieltje's integral.
7. Give an example of an integral which depends on a parameter.
8. State Cartheodory extension theorem.
9. Define a sigma field and show that it is closed under countable intersections.
10. Show that the set of prime numbers is a Lebesgue measurable set.
11. Define absolute continuous measures.
12. Give an example of two singular measures.

(12 × 2 = 12 weightage)

Part B

Answer any **eight** *questions.*
Weightage 2 for each question.

13. What do you mean by an outer measure ? State and prove the sub-additivity property of outer measure.
14. State and prove the mean value theorem.

Turn over

15. Show that a non-negative continuous measurable function can be represented as the lim non-decreasing sequence of non-negative simple functions.

16. If $\{f_n\}$ is a sequence of measurable functions and $g = \lim_{n \to \infty} f_n$, then show that g is a meas function.

17. If f and g are integrable functions, show that $f + g$ is integrable and $\int f + g = \int f + \int g$.

18. Let A be the subset of [0, 1] which consists of all numbers which do not have the digit 5 ap in their decimal expansion. Examine whether A is Lebesgue measurable. If so find Le measure of A.

19. State and prove Minikowski's inequality.

20. Find $\lim_{\theta \to 0} \int_0^\infty (1 + \theta x)^{1/\theta} e^{-\alpha x} dx$. Give the reason.

21. State and prove Fatou's lemma.

22. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$, show that $(f + g) \in \mathcal{R}(\alpha)$ and $\int_a^b (f + g)\, d\alpha = \int_a^b f\alpha +$

23. State and prove Hahn decomposition theorem.

24. Distinguish between Lebesgue measure and Lebesgue-Stieltjes measure.

(8 × 2 = 16 w

Part C

Answer any **two** *questions.*
Weightage 4 for each question.

25 (a) State and prove Weistrass theorem.

(b) If $\{f_n\}$ and $\{g_n\}$ are two sequences of functions converges uniformly on a set A, s $\{af_n + bg_n\}$ converges uniformly on A, where a and b are real constants.

26. (a) State and prove Holder's inequality.

(b) State and prove Lebesgue dominated convergence theorem.

27. (a) State and prove Jordan decomposition theorem.

(b) Consider the signed measure v defined on the Borel field of subsets of R such that for each Borel set B, $v(\mathrm{B}) = \int_{\mathrm{B}} \left(x^2 - 1\right) e^{-x^2}\, dx$. Write down a positive set and a negative set with respect to v. Also obtain a Hahn decomposition of R.

28. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$ then show that :

(a) fg and $|f|$ are $\in \mathcal{R}(\alpha)$.

(b) $|f| \in \mathcal{R}(\alpha)$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$.

(2 × 4 = 8 weightage)