16P105 (Pages:2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C05 – DISCRETE MATHEMATICS

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

(Short Answer Questions)

Answer **all** Questions

Each question carries 1 Weightage

1. Define connectivity of a graph. Also find the connectivity of $K_{3,4}$
2. Show that dual of K_4 is K_4.
3. Give an example of an Eulerian graph with 5 vertices and 7 edges.
4. Define directed graph with an example.
5. Determine which complete bipartite graphs are complete graphs.
6. Find the connectivity of $K_{3,4}$.
7. Define a partial ordering on a set X.
8. Define a Boolean algebra.
9. Define upper bound and lower bound on a partially ordered set.
10. Prove that every tree with at least two vertices has at least two leaves.
11. Define nfa.
12. State Jordan Curve Theorem.
13. Define lattice with an example.
14. What are Regular languages? **(14 x 1 = 14 Weightage)**

Part B

(Short Essay Questions)

Answer ***any seven*** from the following ten questions (15 - 24)

Each question carries 2 Weightage

15. From the definition of isomorphism, prove that $G \cong H$ if and only if $\overline{G} \cong \overline{H}$.
16. Draw Petersen graph. Also show that Petersen graph is not bipartite.
17. If G is a simple graph, then K(G)≤ $K'(G)$ ≤δ(G).

18. Prove that K_4 and $K_{3,4}$ are not planar.
19. Let X be a finite set and $\leq$ be a partial order on X. Also R is a relation on X defined by xRy if and only if y covers $x(w.r.t. \leq)$. Show that $\leq$ is generated by R.
20. Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra, specifically, to the power set Boolean algebra of the set of all its atoms.
21. Write the following Boolean functions in their disjunctive normal forms:
    (i) $f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$.
    (ii) $g(a, b, c) = (a + b + c)(a' + b + c')(a + b' + c')(a' + b' + c')(a + b + c')$.
22. Explain transition graph of a dfa with an example.
23. Show that $L = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$ is regular.
24. What do you mean by equivalent grammars? Explain with example.

(7 x 2 = 14 Weightage)

Part C (Essay Questions)

Answer any **two** from the following four questions (25-28).
Each question carries 4 weightage

25. Prove that the complete graph K_n can be expressed as the union of k bipartite graphs if and only if $n \leq 2^k$.
26. (i) State and prove Euler's formula.
    (ii) Prove that a graph is bipartite if and only if it has no odd cycle.
27. State and prove Structure Theorem for Boolean functions.
28. Show that the grammar G with $\sum = \{a, b\}$ and productions $S \to SS, S \to \lambda, S \to aSb, S \to bSa$ generates the language L={w: $n_a(w) = n_b(w)$}.

(2 x 4 = 8 Weightage)

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