FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C02 – ANALYTICAL TOOLS FOR STATISTICS - I

(Statistics)

(2015 Admission Onwards)

Time: Three Hours　　　　　　　　　　　　　　　　　Maximum: 36 Weightage

Part A (Answer *all* questions)

1. Define directional derivatives of a multivariable function
2. State Inverse function theorem.
3. Show that the function $f(x,y) = 2x^4 - 3x^2 y + y^2$ has neither a maximum nor a minimum at the origin.
4. What is harmonic function.
5. State Cauchy Integral formula.
6. Write the fundamental theorem of integral calculus
7. Define odd and even functions. Give examples
8. If $\mathcal{L}\{f(t)\} = F(s)$, then find $\mathcal{L}\{e^{-at}f(t)\}$
9. State Fourier integral theorem
10. Define a periodic function and give an example
11. Mention the use of inverse Laplace Transforms
12. Find out the zeros and discuss the nature of singularities of $f(z) = \frac{z-1}{z^3 - 2z}$

(12 x 1=12 weightage)

Part B (Answer any *eight* questions)

13. If $v = tan^{-1}\left(\frac{x^2+y^2}{xy}\right)$ prove that $xv_x + yv_y = \sin 2v$
14. Show that the function $f(x,y,z) = (y+z)^2 + (z+x)^2 + xyz$ has no maximum or minimum value
15. Let $f(z) = \bar{z}e^{-|z|^2}$. Determine the points at which $f'(z)$ exists and find $f'(z)$ at these points.
16. State and prove Liouville's theorem
17. Distinguish between simply and multiply connected domain.
18. Show that the function $u(x,y) = e^{-x}siny$ is harmonic and find the corresponding analytic function.
19. State and prove Morera's theorem.
20. Does the function $f(z) = \frac{1}{z}$ have an antiderivative on the set of all $z \neq 0$. Explain.
21. Find the Laplace transform of : (i) $2\cos 4t$ (ii) e^{2t-3}
22. Find the inverse Laplace transform of $\frac{5s}{(s^2+4)(s^2+9)}$
23. Show that $\int_0^\infty \frac{\cos \lambda x}{\lambda^2+1} d\lambda = \frac{\pi}{2} e^{-x}, x \geq 0$
24. Find the Fourier transform of $F(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$

(8 x 2=16 weightage)

Part C (Answer any *two* questions)

25. State and prove Cauchy's Residue theorem

26. State and prove the Poisson's integral formula.

27. Solve the differential equation by the method of Laplace transform:

$$Y'' - tY' + Y = 1, \; Y(0) = 1, \; Y'(0) = 2 \,.$$

28. Show that   a) $\int_0^\pi \dfrac{\cos 2\theta}{1+a^2-2a\,\cos\theta}\, d\theta = \dfrac{\pi}{1-a^2} \,, \; a^2 < 1$

               b) $\int_{-\infty}^{\infty} \dfrac{dx}{x^2+x+1} = \dfrac{2\pi}{\sqrt{3}} \,.$

(2 x 4=8 weightage)

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