16P153 (Pages:2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C01 – MEASURE THEORY AND INTEGRATION

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

(Answer *all* questions. Weightage 1 for each question)

1. Define Point-wise and Uniform convergence.
2. If $f \epsilon R(\alpha), g \epsilon R(\alpha)$ on $[a, b]$ Show that $(f - g) \epsilon R(\alpha)$
3. Show that the set of prime numbers is a Lebesgue measurable set
4. Define σ field and show that it is closed under countable intersections.
5. Distinguish between simple function and continuous function.
6. If 'f' is integrable, show that $|f|$ is integrable.
7. Show that a set A is a measurable if and only if its indicator function is a measurable function.
8. If f is measurable function and if $f = g$ almost everywhere. Show that g is measurable.
9. Define Lp space
10. State Holder's inequality.
11. State Hahn decomposition theorem.
12. What do you mean by a product space?

(12 x 1=12 weightage)

Part B

(Answer any *eight* questions. Weightage 2 for each question)

13. State and prove Fundamental Theorem of Calculus.
14. Find the necessary and sufficient condition for the uniform convergence of a sequence of function $\{f_n\}$ defined on [a, b].
15. If $f \epsilon R(\alpha), g \epsilon R(\alpha)$ on $[a, b]$ Show that $(f + g) \epsilon R(\alpha)$ and $\int_a^b (f + g)\, d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha$
16. State and prove Fatou's lemma.
17. Define measurable function. If f & g are measurable functions show that $f - 2g$ and f/g, such that $g \neq 0$ are measurable function.
18. Show that the sum and product of two simple functions are simple functions.
19. Distinguish between Lebesgue measure and Lebesgue-Stieltjes measure
20. State and prove Jordan Decomposition theorem
21. Show that convergence in Lp implies convergence in measure.
22. Show that a function f is measurable if and only if f^+ and f^- are measurable functions

23. State Fubini's theorem and point out its applications in Statistics.
24. Show that the sequence $\{f_n\}$ given by $f_n(x) = tan^{-1}nx, x \geq 0$ is uniformly convergent in any interval $[a, b]$, a > 0 and is not uniformly convergent in $[0, b]$

(8 x 2=16 weightage)

Part C

(Answer any two questions. Weightage 4 for each question)

25. If $f \in R(\alpha), g \in R(\alpha)$ on $[a, b]$ Show that
(a) fg and $|f| \in R(\alpha)$
(b) $|f| \in R(\alpha)$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$

26. (a) Prove that if $\lim_{n \to \infty} f_n(x) = f(x)$ then $\lim_{n\to\infty} \int_A f_n = \int_A f$ where A is any measurable subset of [a,b] with finite measure and $\{f_n\}$ is an increasing sequence of measurable function.
(b State and prove Lebesgue Dominated Convergence Theorem.
27. (a) Let $\{f_n\}$ be a sequence in Lp which converges almost everywhere to a measurable function 'f' then prove that $\{f_n\}$ converges in Lp to 'f' by stating necessary conditions.
(b) Let $\mu(X) < \infty$ and $f_n \in Lp$ and $f_n \to f$ uniformly. Then prove that $f \in Lp$ and $f_n \to f$ in Lp
28. State and prove Radon-Nikodym theorem.

(2 x 4=8 weightage)

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