15P155 Name…………………….

Reg. No…………………..

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEB. 2016

(2015 Admissions)

CC15P ST1 C03– Analytical Tools for Statistics II

(STATISTICS)

Time: 3 Hrs. **Max. Weightage: 36**

Part A

(Answer *all* questions. Weightage 1 for each question)

1. Define Vector space and Subspaces.
2. Explain (a) Idempotent matrix (b) Nilpotent matrix.
3. Define rank of a matrix. If A and B are two matrices. Show that rank (AB)= min [rank(A), rank(B)].
4. What is Basis and Dimension of a vector space ?
5. What you mean by linear transformation on Vector spaces?
6. Define g-inverse.
7. Define positive definite matrices.
8. State Rank Nullity theorem.
9. Explain Minimal Polynomial.
10. State Cayley Hamilton theorem.
11. What is Characteristic root of a matrix.
12. Define Orthogonal matrix.

$(\mathbf{12 \times 1 = 12}$ **Weightage**$)$

Part B

(Answer any *eight* questions. Weightage 2 for each question)

13. Let V be a vector space over a Field K, then prove that the intersection of any two Subspaces of V are again a Subspaces of V if and only if one contains another.
14. Check for linear independence and dependence of the following set of vectors V_1=(2, 0, 1, 1), V_2=(1, 4, 1, 1) and V_3=(1, 2, 2, 2)
15. Describe Inner Product space. Give an example.
16. Show that geometric multiplicity cannot exceed algebraic multiplicity.
17. Find Eigen values of $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.
18. Show that similar matrices have same minimal polynomial.
19. Describe the method of finding inverse.
20. Let V be a vector space with dimension n. Show that any linear independent set in V can be extended to a basis of V.
21. Write a short note on Gram–Schmidt process.

22. Find inverse of $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 2 & 4 & 8 \end{bmatrix}$

23. Define the Rank and Signature of a real quadratic forms.

24. Show that Hermitian matrix A is positive definite if and only if each of the principal minors of A is positive.

$(\mathbf{8 \times 2 = 16\ Weightage})$

Part C

(Answer any two questions. Weightage 4 for each question)

25. a) Define Moore Penrose inverse of a matrix. Prove that it is unique.
    b) Define geometric and algebraic multiplicity. Find geometric and algebraic multiplicity of the matrix A=$\begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}$.

26. a) Show that a set of non null vectors α_1, α_2, ...α_n orthogonal in pairs is necessarily independent.
    b) Find Moore Penrose g-inverse of $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$.

27. a) Define quadratic forms, Illustrate different forms of them.
    b) Classify the quadratic form $12x^2+2y^2+3z^2-2yx-8-yz-6xy$

28. a) Let $f = R^3 \rightarrow R^2$ be defined by f(x,y,z)=(z+x, 2x+y). Show that f is linear mapping.
    b) Show that characteristic roots of a skew symmetric matrix are either zero or a pure imaginary number.

$(\mathbf{2 \times 4 = 8\ Weightage})$

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