16P103 (Pages:2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C03 – REAL ANALYSIS-I

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

(Short Answer Questions)

Answer **all** questions.

Each question has 1 weightage.

1. Define a Perfect set. Give an example of a perfect set which is bounded but not open.
2. Show that the set of all rational numbers is not connected.
3. Prove that the set of rational numbers is countable.
4. Prove that every neighborhood is an open set.
5. Let f be a continuous real function defined on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Show that $Z(f)$ is closed.
6. Explain discontinuities of first and second kind.
7. Let f be a differentiable function on $[a, b]$ such that $f'(x) = 0$ for all $x \in (a, b)$. Prove that f is a constant.
8. State Generalized Mean Value Theorem.
9. Let f be a function on $[a, b]$ such that $|f| \in \mathcal{R}$ on [a,b]. Is $f \in \mathcal{R}$ on $[a, b]$? Justify your answer.
10. Let f be continuous on [a,b].Then prove that $f \in \mathcal{R}(\alpha)$.
11. Define rectifiable curves.
12. Let $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}$ where $n = 1,2,3,4, \dots \dots$ and $x \in [0,1]$. Show that $\{f_n\}$ is not equicontinuous family of functions.
13. Define uniform convergence.
14. If $\{f_n\}$ is a sequence of continuous functions defined on E and $f_n \to f$ uniformly on E.Then prove that $\{f\}$ is continuous on E.

(14 x 1 = 14 Weightage)

Part B

Answer any ***seven*** from the following ten questions (15-24).

Each question has weightage 2.

15. Prove that a set E is open if and only if its complement is closed.
16. Construct a bounded set of real numbers with exactly three limit points.
17. For any $x = (x_1, x_2), y = (y_1, y_2)$, define $d(x, y)$ =min$\{|x_1 - y_1|, |x_2 - y_2|\}$. Is d a metric on $\mathbb{R}^2$.

18. Give an example to show that Mean value theorem need not true for complex valued function.
19. If $x_1, x_2, \ldots\ldots, x_k$ are coordinates of the point $x \in \mathbb{R}^k$ and the function φ_i defined by $\varphi_i(x) = x_i$, $(x \in \mathbb{R}^k)$, then prove that φ_i is continuous.
20. Let $[x]$ denote the largest integer less than or equal to x. What type of discontinuities does the function $[x]$ have.
21. Let f be a bounded function and α be a monotonic increasing function on [a,b]. Prove that if f is monotonic on [a,b] then f is Reimann-Steiltjes integrable with respect to α on [a,b].
22. Let f be a bounded function and α be a monotonic increasing function on [a,b] and f^2 is Reimann-Steiltjes integrable with respect to α on [a,b]. Does it imply f is Reimann-Steiltjes integrable with respect to α on [a,b].
23. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2 x}$. For what values of x does the series converges absolutely?
24. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** from the following four questions (25-28).

Each question has weightage 4

25. (a) Prove that every $k\ cell$ is compact.
    (b) If p is a limit point of E, then prove that every neighborhood of p contains infinitely many points of E.
26. (a) Define open mapping. Can you say that continuous function is always open. Justify your answer.
    (b) Let f be monotonic on (a,b).Then prove that the set of points of (a,b) at which f is discontinuous is at most countable.
27. Suppose f is bounded on [a,b], f has only finitely many points of discontinuity on [a,b], and α is continuous at every point at which f is discontinuous,then prove that $f \epsilon \mathcal{R}(\alpha)$.
28. (a) Suppose $\{f_n\}$ is a sequence of functions defined on E,and suppose $|f_n(x)| \le M_n \qquad (x \in E, n = 1,2,3, \ldots\ldots)$. Then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.
    (b) State and prove the Cauchy criterion for uniform convergence of sequence of functions.

(2 x 4 = 8 Weightage)

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