17P162 (Pages:3) Name: ……………………

Reg. No……………………..

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C05 DISTRIBUTION THEORY

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Weightage 1 for each question.

1. For any integer valued random variable X. Show that $\sum_{n=0}^{\infty} s^n P(X \leq n) = \frac{P(s)}{1-s}$ where P(s) is the probability generating function of X.
2. Show that a continuous non-negative random variable X follows the exponential distribution if and only if the relationship $P(X > t + s | X > t) = P(X > s)$ holds for all real $t, s \geq 0$.
3. State recurrence relation for cumulants of power series distribution.
4. Let (X, Y) have the joint probability density function $f(x,y) = \begin{cases} \frac{1}{4} & \text{for } |x|, |y| \leq 1 \\ 0 & \text{elsewhere} \end{cases}$ Find marginal and conditional probability distributions.
5. If and X and Y are independent binomial random variable such that $X \sim B(m,p), Y \sim B(n,p)$. Show that $X/(X+Y)$ is hyper geometric.
6. Given $f(x, y) = 2, 0 < x < y < 1$, Evaluate the conditional expectation E(Y/X)
7. Let X have a standard Cauchy distribution. Find the probability density function of X^2. Identify its distribution.
8. If $X_1, X_2, \ldots, X_n$ are i.i.d r.v's having exponential distribution with pdf $f(x) = \theta e^{-\theta x}$, $x > 0$ and $\theta > 0$. Obtain the distribution of $X_{(n)} = \min(X_1, X_2, \ldots, X_n)$.
9. Express the distribution function of B(n, p) variable in the form of an incomplete beta integral.
10. Find the characteristic function of Laplace distribution and obtain the cumulants.
11. If X and Y are independent Cauchy random variables with X_k distributed $C(\mu_k, \theta_k), k = 1,2$. Show that $X + Y$ is $C(\mu_1 + \mu_2, \theta_1 + \theta_2)$ random variables.
12. If X and Y are independent and identically distributed geometric random variables, Find the distribution of Max(X, Y)

(12 x 1=12 weightage)

Turn over

PART B

Answer any ***eight*** questions. Weightage 2 for each question.

13. Define the Hyper geometric distribution. Show that under conditions the hypergeometric distribution tends to the binomial distribution.

14. Let X_i follows gamma density with with parameters η_i and λ for $i = 1,2$. Assume that X_1 and X_2 are independent. Find the joint distribution $Y_1 = X_1 + X_2$ and $Y_1 = \frac{X_1}{X_2}$.

15. Let (X, Y) be a bivariate normal random variables with parameters $\mu_1, \mu_2, \sigma_1{}^2, \sigma_2{}^2$ and ρ. Let $U = aX + b, a \neq 0$, and $V = cY + d,\ c \neq 0$. Find the joint distribution of (U, V).

16. Define multinomial distribution and obtain it's characteristic function. If $(X_1, X_2, \dots, X_{k-1})$ is distributed as multinomial, Show that the correlation coefficient between X_i and X_j is negative.

17. Let $(X_1,\ X_2, \dots, X_n)$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. Show that $\overline{X}$, and $(X_1 - \overline{X}, X_2 - \overline{X}, \dots,\ X_n - \overline{X})$ are independent.

18. Define Pearsonian family of distributions. Show that gamma and beta distributions are members of this family.

19. For the Pareto distribution specified by $f(x) = \begin{cases} \frac{\beta\alpha^\beta}{x^{\beta+1}} & \text{if } x \geq \alpha \\ 0 & \text{if } x < \alpha \end{cases}$, Show that moment of order n exists only if $n < \beta$. Justify the use of Pareto model as a suitable model in the study of skewed data such as, the distribution of income.

20. If $\mathbf{X} = (X_1, X_2, \dots, X_p)$ is distributed is distributed as multivariate normal $N(\mu, \Sigma)$, Derive the distribution of $Y = \mathbf{CX}$, where $\mathbf{C}$ non- singular.

21. If $X \sim F(m, n)$, Show that $Y = \frac{1}{1+\left(\frac{m}{n}\right)X}$ follows $B(\frac{n}{2}, \frac{m}{2})$.

22. Define the log normal distribution. Also determine mode and median of the distribution.

23. Let $(X_1, X_2, \dots, X_n)$ be i.i.d with PDF $(y) = \begin{cases} y^\alpha, \text{if } 0 < y < 1 \\ 0, \text{otherwise} \end{cases}, \alpha > 0$. Show that $\{X_{(i)} | X_{(n)}, i = 1,2, \dots, (n-1)\}$ and $X_{(n)}$ are independent.

24. If X_1 and X_2 are independent $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ r.v's. Show that $X_1 - X_2$ and $X_1 + X_2$ are independent.

(8 x 2=16 weightage)

PART C

Answer any ***two*** questions. Weightage 4 for each question.

25. In sampling from normal population, show that the sample mean $\overline{X}$ and sample variance S^2 are independently distributed.

26. (a). State Chebyche'v inequality. If X be distributed with pdf f(x) = 1, 0 < x < 1 prove that $P\left(\left|X-\frac{1}{2}\right|<2\sqrt{\frac{1}{2}}\right) \geq \frac{3}{4}$

(b). A random sample of size n is taken from a population with distribution $f(x)=\begin{cases} e^{-x}, & \text{if } x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$ Find the probability density function of the range.

27. (a). Obtain the characteristic function of the multivariate normal distribution and establish the reproductive property.

(b). If $X_1, X_2, \ldots, X_n$ are i.i.d random variables following $N(\mu, \sigma^2)$. Obtain the distribution of $L = \sum_{i=1}^{n} l_i X_i$ where l_i's are constants. What can you say about independence of linear forms?

28. Define Non central Chi-square distribution and derive its probability density function. Also find the expression for the mean and variance.

(2 x 4=8 weightage)

****