17P101 (Pages:2) Name: ……………………

Reg. No……………………

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C01/ CC17P MT1 C01 - ALGEBRA- I

(Mathematics)

(2015 Admission Onwards)

Time: Three hours Maximum: 36 Weightage

PART A

*Answer **all** the questions. Each question carries 1 weightage*

1. Describe all symmetries of a line segment in R.
2. Find the order of (8,10) in $Z_{12} \times Z_{18}$.
3. Show that for word addition of binary words u and v of the same length u + v = u – v.
4. Find the order of $Z_4 \times Z_{12}/(\langle 2\rangle \times \langle 2\rangle)$.
5. Describe the center of every simple abelian group.
6. Can an infinite abelian group have composition series? Justify your answer.
7. Find the number of orbits in {1,2,3,4,5,6,7,8} under the cyclic subgroup $<(1\,3\,5\,6)>$ of S_8.
8. Show that if H and N are subgroups of G, and N is normal in G then $H \cap N$ is normal in G.
9. Find all sylow 3- subgroups of S_4 and show that they are conjugate.
10. Find the reduced form and the inverse of the reduced form of $a^2 a^{-3} b^3 a^4 c^4 c^2 a^{-1}$.
11. What is group presentation?
12. Find all zeros of $x^3 + 2x + 2$ in Z_7 with coefficients in Z_7.
13. Determine whether $8x^3 + 6x^2 - 9x + 24$ in Z[x] satisfies an Eisenstein criterion for Irreducibility Over Q.
14. Find a subring of the ring Z × Z that is not an ideal of Z × Z.

(14 x 1=14 weightage)

PART B

*Answer **any seven** questions. Each question carries 2 weightage*

15. Find all abelian groups up to isomorphism of order 720.
16. Show that if G has a composition series and if N is a proper normal subgroup of G, then there exist a composition series containing N.

17. Consider the (6,3) linear code C with the standard generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Give the parity check equations for this code and list the code words in C.

18. Show that, if a finite group G contains a proper subgroup of index 2 in G,then G is not simple.

19. Let X be a G-set and let $Y \subseteq X$.Let $G_Y = \{g \in G/gy=y \text{ for all } y \in Y\}$.Show G_Y is a subgroup of G

20. Find the number of distinguishable ways the edges of a square of cardboard can be painted if six colors of paint are available and

   a. No color is used more than once

   b. The same color can be used on any number of edges.

21. Let K and L be normal subgroups of G with K v L = G,and $K \cap L = \{e\}$. Show that $G/K \cong L$ and $G/L \cong K$.

22. Show that there are no simple groups of order 255 = (3)(5)(17).

23. Show that $(a, b : a^3 = 1, b^2 = 1, ba = a^2 b)$ gives a group of order 6. Show that it is non- abelian

24. Show that the polynomial $x^P + a$ in $Z_P[x]$ is not irreducible for any $a \in Z_P$.

(7 x 2=14 weightage)

PART C

Answer any two. Each carries 4 weightage

25. State and prove Eisenstein's theorem. Using it prove that the cyclotomic polynomial: $\varphi_p(x) = \frac{x^P - 1}{x - 1} = x^{p-1} + x^{p-2} + \ldots\ldots + x + 1$ is irreducible over Q for any prime p.

26. State and prove first Sylow theorem. Show that a normal p- subgroup of a finite group is contained in every sylow p- subgroup of G.

27. Let X be a G-set and let $x \in X$. Show that G_x is a subgroup of G and that $|Gx| = (G : GX)$.

28. State and prove second isomorphism theorem. Use this to show that a subgroup K of a solvable group G is solvable.

(2 x 4=8 weightage)

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