17P102 (Pages:2) Name: ....

Reg. No.....................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1C02/ CC17P MT1C02 – LINEAR ALGEBRA

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Maximum:36 Weightage

PART - A

Answer ***all*** questions. Each question carries 1 weightage

1. Let V be a vector space over a field F and $0 \in F$. Prove that if $c.v = 0$, where $v \in V$ and $c \in F$, then either $c = 0$ or $v = 0$.
2. Let V be the vector space of all real valued functions on $\mathbb{R}$. Verify whether $W = \{f \in V: f(x^2) = (f(x))^2\}$ is a subspace of V.
3. Find the dimension of the space of all upper triangular matrices of order 3×3 over $\mathbb{R}$.
4. Find the co-ordinate matrix of the vector $(1, 0, 1)$ in the basis of $\mathbb{C}^3$ consisting of the vectors $(2i, 1, 0), (2, -1, 1), (0, 1 + i, 1 - i)$ in that order.
5. Verify whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x^2, y)$ is a linear transformation.
6. Find the nul lspace of linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x - 3y, 0)$.
7. Write any nonzero linear functional on the space of all $n \times n$ matrices over the field $\mathbb{R}$.
8. Let $W = \text{span}\{(1, 0, 1), (1, 1, 0)\}$. Find a nonzero linear functional in W°.
9. Find the characteristic values of $\begin{bmatrix} 3 & 4 \\ 3 & -1 \end{bmatrix}$.
10. Let W span $\{(1, 2, 1)\}$ in $\mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x + y, 3y, y + z)$. Verify whether W is an invariant subspace of T.
11. Let $W_1 = \text{span}\{(1, 0, 0), (0, 1, 1)\}$ and $W_2 = \text{span}\{(2, 1, 1)\}$. Verify whether $W_1 + W_2$ is a direct sum.
12. Verify whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, 0)$ is a projection.
13. Write any vector in the orthogonal complement of $W = \{(x, 2x): x \in \mathbb{R}\}$.
14. Prove that if innerproduct $(\propto |\beta) = 0$ for all β, then $\propto = 0$.

(14 × 1 = 14 weightage)

PART -B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Let W_1 and W_2 are subspaces of a vectospace V, then Prove that $W_1 \cup W_2$ is also a subspace if and only if one of the subspaces W_i is contained in the other.

16. Let V be a finite dimensional vector space with dimension n, then prove that any subset of V which contains more than n vectors is linearly dependent.`
17. Find the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, y, x - z)$ with respect to the ordered basis $\mathfrak{B} = \{(1, 0, 1), (1, 1, 1), (2, 0, 1)\}$.
18. Let f be a nonzero linear functional on a vector space V, then prove that every hyperspace in V is the null space of a nonzero linear functional on V.
19. Let V be a vector space over the field F and let T be a linear transformation from V into V. Prove that the following are equivalent
    (a) The intersection of the range of T and the nullspace of T is the zero subspace of V.
    (b) If $T(T\alpha) = 0$, then $T\alpha = 0$.
20. Let T be a linear transformation from V into W. Prove that T is nonsingular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.
21. Prove that similar matrices have the same characteristic polynomial.
22. Let V and W be finite dimensional vector spaces over the field F and let T be a linear transformation. Prove that *(1)* nullspace of T^t is the annihilator of the range of T. *(2)* $\text{rank}(T^t) = \text{rank}(T)$.
23. Let W be a finite dimensional subspace of an innerproduct space V and E be the orthogonal projection of V onto W. Prove that *(1)* E is an idempotent linear transformation of V onto W *(2)* $W^\perp$ is the nullspace of E.
24. Prove that the set of orthogonal vectors are linearly independent.

(7×2=14 weightage)

PART -C

Answer any ***two*** questions. Each question carries 4 weightage.

25. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then prove that $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.
26. If V is a finite dimensional vector space over a field F with dimension n, then prove that $\dim L(V, V) = n^2$.
27. State and prove Cayley Hamilton theorem.
28. Explain Gram-Schmidt orthogonalization process and find an orthonormal basis for $\mathbb{R}^3$ with standard inner product by applying this method to the vectors (3, 0, 4), (-1, 0, 7), (2, 9, 11).

(2×4=8 weightage)

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