17P158 (Pages:2) Name: ……………………

Reg. No. ……………………

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15PST1C01 – MEASURE THEORY AND INTEGRATION

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum:36 Weightage

PART A

Answer ***all*** questions. Weightage 1 for each question.

1. Define Riemann-Stieltjes integral.
2. State Second mean value theorem.
3. Examine whether the function $f(x) = \frac{1}{nx+1}, 0 < x < 1, n = 1,2, \dots\dots$ is converging uniformly to zero.
4. Give an example for a normed linear space.
5. What is Banach space?
6. State Fatou's lemma.
7. Show that $\int (f+g)d\mu = \int f d\mu + \int g d\mu$, where f, g $\in M^+$.
8. Define convergence in L_p .
9. State monotone class lemma.
10. If f is a measurable function, show that $| f |$ is also a measurable function.
11. State Radon-Nikodym theorem.
12. Define outer measure.

(12 ×1 = 12 Weightage)

PART B

Answer any ***eight*** questions. Weightage 2 for each question.

13. State and prove the fundamental theorem of integral calculus.
14. Suppose F and G are differentiable functions on [a,b], $F' = f \in R$, and $G' \in R$, then

    $\int F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$.
15. Show that a measurable function $f \in L$ if and only if $| f | \in L$ and hence

    $| \int f \, d\mu | \leq \int | f | \, d\mu$.
16. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a,b], then show that

    (i) $fg \in R(\alpha)$ and (ii) $|f| \in R(\alpha)$.
17. Define almost uniform convergence. If $\{f_n\}$ is a sequence of functions which converges almost uniformly to f, then show that $\{f_n\}$ converges in measure to f.

18. Define Cauchy sequence. Prove that every convergent sequence is a Cauchy sequence.

19. Suppose that $f \in M^{+}(X, \mathbb{A})$. Then $f(x) = 0$ μ-almost everywhere on X iff $\int f d\mu = 0$.

20. State and prove Hahn – decomposition theorem.

21. Show that characteristic function of a set A is measurable, iff the set A is measurable.

22. State and prove Fubini's theorem.

23. Establish Holder's Inequality.

24. Show that the outer measure is countably subadditive.

(8×2 = 16 Weightage)

PART C

Answer any ***two*** questions. Weightage 4 for each question.

25. (a) State and prove Stone-Weierstrass theorem.

(b) Show that the sequence $\{f_n(x)\}$ where $f_n(x) = tan^{-1} nx$, $x \geq 0$ is uniformly convergent on in any interval [a,b], a > 0.

26. (a) State and prove the necessary and sufficient condition for a function to be Riemann-Stieltjes integral.

(b) Show that if $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$ then $f \in R(\alpha_1 + \alpha_2)$.

27. (a) State and prove Lebesgue Decomposition theorem.

(b) Show that the Lebesgue space $L_1(X, \mathbb{A}, \mu)$ is a normed linear space.

28. (a) State and prove Jordan decomposition theorem.

(b) State and prove the property of absolute integrability.

(4×2= 8 Weightage)

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