

17P105

Pages:2

Name:.....

Reg.No:.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(CUCSS-PG)

CC17P MT1 C04 - NUMBER THEORY

(Mathematics)

(Regular - 2017 Admissions)

Time :Three Hours

Maximum : 36 weightage

Part A

Answer all questions. Each carries 1 weightage

1. Find all integers such that $\phi(n) = \phi(2n)$.
2. Define Möbius function $\mu(n)$ and show that $\sum_{d|n} \mu(d) = 0$ if $n > 1$.
3. Give an example of a multiplicative function which is not completely multiplicative.
4. Assume f is multiplicative, then prove that $f^{-1}(p^2) = (f(p))^2 - f(p^2)$ for every prime p .
5. Define divisor functions $\sigma_\alpha(n)$ for $n \geq 1$ and show that they are multiplicative.
6. Prove that $[x] + [x + \frac{1}{2}] = [2x]$.
7. For $x \geq 2$ prove that $\vartheta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$.
8. Show that if $a > 0$ and $b > 0$, then $\lim_{x \rightarrow \infty} \frac{\pi(ax)}{\pi(bx)} = \frac{a}{b}$.
9. For $x \geq 2$, prove that $\sum_{n \leq x} \frac{n}{\phi(n)} = O(x)$.
10. Calculate highest power of 10 that divides 1000!.
11. If p is an odd prime, prove that $\sum_{r=1}^{p-1} r(r/p) = 0$ if $p \equiv 1 \pmod{4}$.
12. Derive Selberg identity.
13. Write a note on enciphering key.
14. What is a cryptosystem?

(14 × 1 = 14 Weightage)

Part B

Answer any seven questions. Each carries 2 weightage

15. Let f be multiplicative. Then f is completely multiplicative if, and only if, $f^{-1}(n) = \mu(n)f(n)$ for all $n \geq 1$. Thus find the inverse of Euler's function ϕ .
16. Prove that $\sum_{d|n} \mu(d) \log^m d = 0$, if $m \geq 1$ and n has more than m distinct prime factors.
17. Prove that $\sum_{n \leq x} \lambda(n) \left[\frac{x}{n} \right] = [\sqrt{x}]$.
18. State and prove Euler's summation formula.
19. State and prove Abel's identity.
20. Show that the Legendre's symbol (n/p) is a completely multiplicative function of n .
21. Show that if p is an odd positive integer then:
 - (a) $(-1/p) = (-1)^{\frac{p-1}{2}}$
 - (b) $(2/p) = (-1)^{p^2-1/8}$
22. Find the inverse of the matrix $\begin{pmatrix} 15 & 17 \\ 4 & 9 \end{pmatrix} \pmod{26}$.
23. Explain briefly about digraph transformations.
24. Explain the RSA cryptosystem. Illustrate with an example.

(7 × 2 = 14 Weightage)

Part C

Answer any two questions. Each carries 4 weightage

25. Prove that for every integer $n \geq 2$ we have $\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$.
- 26.(a) State and prove the quadratic reciprocity law.
(b) Determine whether 219 is a quadratic residue or nonresidue mod 383.
27. Explain the advantages and disadvantages of public key cryptosystem as compared to classical cryptosystems.
28. State Abel's Identity and deduce Euler's Summation formula from Abel's identity.

(2 × 4 = 8 Weightage)

(14 × 1 = 14 Weightage)