18P103 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C03 / CC17P MT1 C03 – REAL ANALYSIS-I

(Mathematics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that interior of a set is open.
2. Prove that every neighborhood is convex.
3. Show that length of the Cantor set is zero.
4. Is set of irrational numbers connected? Justify your answer.
5. Is every continuous function is an open mapping? Justify.
6. Define simple discontinuity with an example.
7. State intermediate value theorem. Is the converse true?
8. Discuss the differentiability of the greatest integer function in R.
9. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then show that $f\,g \in R(\alpha)$
10. Show that if $f_1(x) \leq f_2(x)$ on $[a, b]$, then $\int_a^b f_1\, d\alpha \leq \int_a^b f_2\, d\alpha$
11. If $\gamma(t) = e^{2it}$ where $0 \leq t \leq 2\pi$. Show that γ is rectifiable.
12. If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1,2,3 \ldots$, and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then show that $\{f_n\}$ contains a uniformly convergent subsequence.
13. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
14. Construct sequences $\{f_n\}$ and $\{g_n\}$ which converge uniformly on some set, but such that $\{f_n\, g_n\}$ does not converge uniformly on E.

(14 × 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Construct a compact set of real numbers whose limit points form a countable set.
16. Prove or disprove: "A set of finite points has no limit point".

17. Let f be monotonic on (a, b). Then show that the set of points of (a, b) at which f is discontinuous is at most countable.
18. Show that if f is continuous mapping of a metric space X into a metric space Y and if E is connected subset of X, then f(E) is connected.
19. State L'Hospital's rule. Is the rule true for complex valued functions? Justify.
20. State and prove Taylor's theorem.
21. If γ' is continuous on $[a, b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$
22. If $f \in R$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that, $F' = f$ then $\int_a^b f(x) dx = F(b) - F(a)$
23. Construct a real continuous function on the real line which is nowhere differentiable.
24. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}$ For what values of x does the series converge absolutely? On what intervals does it converge uniformly? On what intervals does it fail to converge uniformly ?

(7 × 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let P be a nonempty perfect set in R^k , then show that P is uncountable.
26. Prove that monotonic functions have no discontinuities of the second kind.
27. Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then show that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$
28. State and prove Stone – Weierstrass theorem.

(2 × 4 = 8 Weightage)

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