18P163 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C02 – ANALYTICAL TOOLS FOR STATISTICS - I

(Statistics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define limit of a multivariable function.
2. Briefly explain the concept ‘ directional derivative’.
3. Define Riemann integral of a multivariable function.
4. Define an analytic function and give an example.
5. Show that the real part of an analytic function is harmonic.
6. State Cauchy’s Integral formula and evaluate $\int_{|z|=2} \frac{dz}{(z-3)(z-1)}$.
7. What kind of singularity does $\frac{\sin z}{z^2}$ have at $z=0$? Explain.
8. Distinguish between the terms residue at a pole and residue at infinity.
9. Define removable singularity and illustrate with an example.
10. Define Laplace transform of a function. Obtain the same of a constant function.
11. If the Laplace transform of F(t), L{ F(t)}= f(s), then find L{ e^{at} F(t)}.
12. State Fourier integral theorem.

(12 x 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Investigate the continuity of $f(x,y)=\begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0). \end{cases}$
14. Show that $z = x\ \cos(y/x) + \tan(y/x)$ satisfies $x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy} = 0$ except at points for which x = 0.
15. Examine the function $x^2 + y^2 + x + y + xy$ for maximum and minimum.

16. State Cauchy-Riemann equations and verify the same for f(z)=z^2.

17. Find an analytic function whose real part is $u = e^x(x \cos y - y \sin y)$.

18. Evaluate $\int_{|z|=2} \frac{1}{(z - 1/2)^2 (z-1)} dz$.

19. State and prove Moreras theorem.

20. Find Laurents series expansion of $f(z) = \frac{1}{z^2(1-z)}$, specifying the region of expansion.

21. Integrate $\frac{1}{z^4 - 1}$ around the circle (a) $|z+1| = 1$ (b) $|z+3| = 1$.

22. Find the Fourier transform of $f(x) = x \quad if \quad a \le x \le b$.

23. If the Laplace transform of F(t), L{F(t)}=f(s), show that $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$.

24. Define inverse Laplace transform of a function and obtain the same of $\frac{2s^2+1}{s(s+1)^2}$.

(8 x 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. If xyz= abc , find the optimum value of bcx+cay+abz.

26. State and prove the necessary and sufficient condition for a function to be analytic.

27. Establish residue theorem and evaluate $\int_0^{\pi} \frac{1}{2+\cos\theta} d\theta$.

28. Find the Fourier series expansion of $f(x) = x + x^2$, $-\pi < x < \pi$.

    Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ...$

(2 x 4 = 8 Weightage)

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