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FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019 (CUCSS PG)
CC19P MST1 C01 - ANALYTICAL TOOLS FOR STATISTICS - I (Statistics)
(2019 Admission Regular)
Time: Three Hours
Maximum: 30 Weightage

## PART A

Answer any four questions. Each question carries 2 weightage.

1. Verify the Cauchy-Reimann equations of the complex function $f(z)=\frac{\bar{z}}{|z|^{2}}$
2. Show that the real and imaginary parts of the analytic function satisfies Laplace equation.
3. Find the complex function where the real part is given by $u_{x}=e^{x}(\cos (y)-y \sin (y))$ given that the function is analytic.
4. Evaluate $\int_{C} Z^{2} d z$ where $C$ the straight line is joining the origin to the point.
5. State and prove Cauchy's integral formula.
6. Find
a). $\int_{C} \frac{3 Z^{2}+7 Z+1}{Z+1} d z$, where $C:|Z+1|=1$
b). $\int_{C} \frac{5 z-2}{Z(Z-1)} d z$, where $C:|Z|=2$
7. Show that $\lim _{(x, y) \rightarrow(0,0)} f(x, y)$ does not exist, where $f(x, y)= \begin{cases}\frac{x^{2}-y^{2}}{x^{2}+y^{2}}, & (x, y) \neq 0 \\ 0 & (x, y)=0\end{cases}$

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(4 \times 2=8 \text { Weightage })
$$

## PART B

Answer any four questions. Each question carries 3 weightage.
8. Explain the method of Lagrangian multiplier.
9. Find the inverse Laplace transform of $\frac{2 s+7}{3 s^{2}+5}$ and $\frac{1}{s(1+2 s)}$
10. Derive the polar form of Cauchy Reimann equations.
11. Find the Taylor series expansion of
a). $\frac{1}{Z^{2}-3 Z+2}$ in $0<|Z|<1$
b). $\frac{5 z+7}{(Z+2)(Z+3)}$ in $|Z|<2$
12. Find the Fourier transform of $f(x)=e^{-|x|},-\infty<x<\infty$
13. State and prove Laurent's theorem.
14. Find the Laplace transform of the following functions:
a) $t \sin (\beta t)$
b) $\cos (a t)$
c) $t e^{t}+\cosh (t)$
( $\mathbf{4} \times \mathbf{3}=\mathbf{1 2}$ Weightage)

## PART C

Answer any two questions. Each question carries 5 weightage.
15. Solve the initial value problem using Laplace transform

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y^{\prime \prime}-y^{\prime}-6 y=0, y(0)=6, y^{\prime}(0)=13
$$

16. Evaluate $\int_{0}^{\pi} \frac{d \theta}{1+\cos (\theta)}$
17. State and prove Poisson's Integral formula.
18. Find the Fourier series corresponding to the function

$$
f(x)=\left\{\begin{array}{l}
-k, \text { when }-\pi<x<0 \\
k, \text { when } \quad 0<x<\pi
\end{array} \text { and } f(x+2 \pi)=f(x) .\right.
$$

