19P104 (Pages: 2) Name: …….……

Reg. No…….……...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCSS PG)

CC19P MTH1 C04 - DISCRETE MATHEMATICS

(2019 Admission Regular)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Give an example of a poset with no maximum element and with exactly one maximal element.
2. Let X be the set of positive integers which divide 30. Define a relation $\leq$ on X by $x \leq y$ if and only if x divides y. Draw the Hasse diagram of this relation.
3. Let $(X, +, \cdot, ')$ be a Boolean algebra. Prove that $x.(x + y) = x$ for all $x, y \in X$.
4. If a simple graph G is not connected, prove that G^c is connected.
5. Define the normal product of two simple graphs.
6. If the girth k of a connected plane graph G is at least 3, prove that $m \leq \frac{k(n-2)}{k-2}$
7. Find a grammar for $\sum = \{a, b\}$ that generate the set of all strings with at least one a.
8. Find a DFA which accepts the string 11 only.

(8 x 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Let X be a finite set and $\leq$ be a partial order on X. R is a binary relation on X defined by xRy iff y covers x. Prove that $\leq$ is the smallest order relation containing R.
10. If $(X, \leq)$ is a bounded, complemented and distributive lattice, prove that there exist a Boolean algebra structure on X, $(X, +, \cdot, ')$ such that the partial order relation defined by this structure coincides with the given relation $\leq$.
11. Prove that the characteristic numbers of a symmetric Boolean function completely determine it.

UNIT II

12. Show that the connectivity and edge connectivity of a simple cubic graph are equal.
13. Prove that the number of edges in a tree with n vertices is $n - 1$.
14. State and prove Euler's formula for a plane graph.

UNIT III

15. Show that the grammar G with productions S $\longrightarrow$ SS / λ / aSb / bSa generates the language in which every sentence is a string with equal number of a's and b's.
16. Show that the language $L = \{awa : \ w \in \{a, b\}^*\}$ is regular.
17. Design an NFA for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$

(6 x 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove the Stone representation theorem for finite Boolean algebras.
19. a) Show that for any loopless connected graph , $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

    b) Prove that a graph G with at least three vertices is 2-connected if, and only if, any two vertices of G are connected by at least two internally disjoint paths.
20. For a connected graph G, show that the following statements are equivalent.
    a) G is Eulerian
    b) The degree of each vertex of G is an even positive integer.
    c) G is an edge-disjoint union of cycles.
21. a) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a dfa and let G_M be its associated transition graph. Then prove that for every $q_i, q_j \in Q$, and $w \in \Sigma^+, \delta^*(q_i, \mathrm{w}) = q_j$ if, and only if, there is in G_M a walk with label w from q_i to q_j.

    b) Find a DFA that recognises the set of all strings on $\{a, b\}$ starting with prefix ab.

(2 x 5 = 10 Weightage)

****