19P107A (Pages: 2) Name: …….

Reg. No…….

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Supplementary/Improvement)

(CUCSS-PG)

CC15P PHY1 C02/ CC17P PHY1 C02 – MATHEMATICAL PHYSICS - I

(Physics)

(2015 to 2018 Admissions)

Time: 3 Hours Maximum: 36 Weightage

SECTION A

Answer ***all*** questions. Each question carries 1 weightage.

1. Determine $\boldsymbol{\nabla}.\boldsymbol{e_1}$ and $\boldsymbol{\nabla}\times\boldsymbol{e_1}$, if $\boldsymbol{e_1}$ is the unit vector in the direction of increasing q_1
2. Write down the 3-D rotation matrix for the rotation of co-ordinates through an angle θ about the x-axis.
3. If $\mathbf{A}$ is irrotational, show that $\mathbf{A}\times\mathbf{r}$ is solenoidal.
4. Show that trace remains invariant under similarity transformation.
5. Explain the quotient rule of tensors.
6. What are pseudo tensors? Give examples of a pseudo scalar and a pseudo vector.
7. What is the significance of the Gram Schmidt orthogonalisation procedure?
8. Obtain the value of $\Gamma\left(\frac{1}{2}\right)$
9. Derive the orthogonality relation for Legendre polynomials.
10. Show that $H_n'(x) = 2nH_{n-1}(x)$
11. Obtain the Fourier sine and cosine transform of e^{-at}
12. Determine the inverse Laplace transform of $[s^2(s^2+a^2)^{-2}]$

(12 x 1 = 12 Weightage)

SECTION B

Answer any ***two*** questions. Each question carries 6 weightage.

13. Obtain the expression for divergence and curl in orthogonal curvilinear coordinate system. Hence deduce the expression for divergence and curl in cylindrical polar coordinate system.
14. Determine the eigenvalues and normalized eigen vectors of the matrix $H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Also, determine the transformation matrix that shall diagonalize H
15. Obtain the general solution to the Bessel's differential equation. How the second solution technique leads to its most general solution?

16. Find the Fourier series of the function f(x) $= \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. Plot the graph of f(x)

(2 x 6 = 12 Weightage)

SECTION C

Answer any **four** questions. Each question carries 3 weightage.

17. Resolve the Cartesian unit vectors into their spherical polar components.

18. (a) If λ is an eigen value of a matrix A, show that λ^2 is an eigen value of A^2.

(b) Prove that the product of two Hermitian operators is Hermitian if and only if they commute.

19. Show that $\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$

20. Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{(3-x^2)}{x^2} sinx - \frac{3}{x} cosx\right]$

21. Express $\int_0^{\pi/2} sin^p\theta cos^q\theta d\theta$ in terms of gamma function. Using this result, evaluate

$$\int_0^1 \frac{dx}{\left(1-x^n\right)^{1/n}}$$

22. Determine the Laplace transform of $\sqrt{t}$

(4 x 3 = 12 Weightage)

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