19P159A (Pages: 2) Name: ……….……………

Reg. No……….…………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C05 – DISTRIBUTION THEORY

(Statistics)

(2015 to 2018 Admissions)

Time: Three Hours Maximum : 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find probability generating function for the random variable X with geometric distribution $P(X = k) = pq^k\,; k = 0,1,2, \dots, 0 < p < 1$
2. If X ~ N (μ, σ^2) find the distribution of Y = e^x
3. Define negative binomial distribution. Deduce any two distribution from it.
4. Define logistic distribution.
5. Define Pareto distribution. Find the mean and variance of this distribution.
6. When do you say that two random variables X and Y are independent? Prove that if X and Y are independent and a < c, b < d are real numbers, then
$$P(a < X \le c, b < Y \le d) = P(a < X \le c).P(b < Y \le d)$$
7. Suppose X has normal distribution with mean zero and variance 1. Find the distribution of X^2
8. Give the characterization of Weibul distribution.
9. Show that hyper geometric distribution tends to binomial distribution as n → ∞
10. Write a short note on multiple correlations.
11. State and prove lack of memory property of exponential distribution.
12. Show that the sum of independent Chi-square variates is also a Chi-square variate.

$(\mathbf{12 \times 1 = 12\ Weightage})$

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Find the m.g.f of multinomial distribution.
14. Find the m.g.f of standard binomial variate $\frac{X-np}{\sqrt{npq}}$ and obtain its limiting form as $n \to \infty$. Also interpret the result.
15. Let two independent r.v's X_1 and X_2 have the same geometric density. Find the conditional distribution of $X_1 \mid X_1 + X_2$
16. If X has a standard Cauchy distribution, find the distribution of $Y = |X|$

17. Let X_1 and X_2 be independent random variable with p.d.f 's $f_1(x) = \frac{\alpha^{p_1}}{\Gamma p_1} e^{\alpha x_1} x_1^{p_1 - 1}$ and $f_2(x) = \frac{\alpha^{p_2}}{\Gamma p_2} e^{\alpha x_2} x_2^{p_2 - 1}$ respectively. Show that $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1 + X_2}$ are independent. Also find the distribution of Y_1 and Y_2

18. If $X \sim N_p$ (μ, Σ). Show that $Y = CX \sim N_p$ (cμ , $c\Sigma c'$), where C is non-singular.

19. Define log-normal distribution. Explain its relationship with normal distribution. Also find its mean and variance.

20. Two dimensional r. v (X, Y) have the joint density

$$f(x, y) = \begin{cases} kx(x - y)\,;\, 0 \le x \le 2, -x < y < x \\ 0; \qquad\qquad\qquad \text{otherwise} \end{cases}$$

Find the conditional probability density function of Y given X. Also find the probability of the event A: {X: x ≥ 1}

21. If X follows uniform distribution with $(x) = x$, $0 \le x \le 1$, then show that $Y = -2 \log X$ is exponential.

22. If X has a Beta (m, n) distribution, then show that $Y = \frac{m}{n} \frac{X}{(1-X)}$ has $F(2m, 2n)$ distribution.

23. Show that $\bar{X}$ and S^2 are independently distributed and obtain the distribution of S^2

24. Derive the p.d.f of F- distribution.

$(\mathbf{8 \times 2 = 16\ Weightage})$

Part B

Answer any ***two*** questions. Each question carries 2 weightage.

25. (a) Establish recurrence relation satisfied by cumulants of power series distribution.

(b) Define generalized Laplace distribution. Obtain its r^th raw moment.

26. Define multivariate normal distribution. Find its mean and variance. Also derive the characteristic function of this distribution.

27. (a) Let $X_{(1)}$, $X_{(2)}$,…, $X_{(n)}$ be the set of order statistic of independent r.v's $X_1, X_2, \ldots, X_n$ with common p.d.f $f(\theta) = \beta e^{-\beta x}$; x > 0. Find the p.d.f of $X_{(r+1)} - X_{(r)}$

(b) Write down the differential equation which generates the Pearsonian distribution. Determine the constants of the equation in terms of moments.

28. Define the non-central t-statistic and derive the probability distribution. When will this reduce to the central 't' distribution? Mention some applications of this statistic.

$(\mathbf{2 \times 4 = 8\ Weightage})$

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