C 63092 (Pages : 3) Name..............................

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Mathematics

MT 2C 09—PDE AND INTEGRAL EQUATIONS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries 1 *weightage.*

1. Determine a partial differential equation satisfied by all surfaces of revolution with z-axis as the axis of revolution.

2. Show that $2z = (ax+y)^2 + b$ is a complete integral of $px + qy - q^2 = 0$.

3. Find the integral of the equation $y\,dx + x\,dy + 2z\,dz = 0$.

4. Determine the region D in which the two equations $xp - yq - x = 0$ and $x^2 p + q - xz = 0$ are compatible.

5. Determine the Monge cone in the case of $p^2 + q^2 = 1$ with vertex (0, 0, 0).

6. Show that if $f(z) = u(x,y) + i\,V(x,y)$ is analytic in $z = x + iy$, then u and v satisfy Laplace's equation in two variables.

7. What is the domain of dependence in the case of a one-dimensional wave equation ?

8. State the Neumann problem.

9. Show that the solution to the Dirichlet problem is stable.

10. State Harnack's theorem.

11. Show that if $y''(x) = F(x)$ and y satisfies the end conditions $y(0) = 0$ and $y(1) = 0$, then :

$$y(x) = \int_0^1 K(x,\xi)\,F(\xi)\,d\xi, \text{ where :}$$

$$K(x,\xi) = \begin{cases} \xi(x-1) \text{ when } \xi < x \\ x(\xi - 1) \text{ when } \xi > x \end{cases}.$$

Turn over

12. Show that the characteristic numbers of a Fredholm equation with a real symmetric Kernel are all real.

13. Show that the Kernel $K(x,\xi) = 1 + 3x\xi$ has a double characteristic number associated with (–1, 1), with two independent characteristic functions.

14. Determine the resolvent Kernel associated with $K(x,\xi) = \cos(x+\xi)$ in $(0, 2\pi)$, in the form of a power series in λ.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. Find the general integral of the equation $(y+1)p + (x+1)q = z$.

16. Explain Charpit's method to find a complete integral of the equation $f(x, y, z, p, q) = 0$.

17. Find a complete integral of the equation $p^2 x + q^2 y = z$ by Jacobi's method.

18. Find the integral surface for the differential equation :
$z(x_{z_x} - y_{z_y}) = y^2 - x^2$ passing through $(2s, s, s)$.

19. Obtain D'Alembert's solution which describe's the vibrations of an infinite string.

20. Reduce the equation $u_{xx} - 4x^2 u_{yy} = \frac{1}{x} u_x$ into Canonical form.

21. Solve : $u_t = u_{xx}, 0 < x < l, t > 0$

$$u(0,t) = u(l,t) = 0$$

$$u(x,0) = x(l-x), 0 \le x \le l.$$

22. Transform the problem :

$$\frac{d^2y}{dx^2} + y = x, \; y(0) = 1, \; y'(1) = 0$$

to a Fredholm integral equation

23. Solve the Fredholm integral equation by iterative method :

$$y(x) = \lambda \int_0^1 x \, \xi \, y(\xi) \, d\xi + 1.$$

24. Write a short note on Neumann series.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*

Each question carries 4 weightage.

25. Show that a necessary and sufficient condition that the Pfaffian differential equation :

$$\vec{X} \cdot d\vec{r} = P(x, y, z) \, dx + Q(x, y, z) \, dy + R(x, y, z) \, dz = 0$$

be integrable is that $\left(\vec{X} \cdot \text{curl } \vec{X}\right) = 0$.

26. Find the solution of the equation :

$$z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y),$$

which passes through the x-axis.

27. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

28. (a) Show that the characteristic values of λ for the equation :

$$y(x) = \lambda \int_0^{2\pi} \sin(x + \xi) \, y(\xi) \, d\xi$$

are $\lambda_1 = 1/\pi$ and $\lambda_2 = -1/\pi$, with corresponding characteristic functions of the form $y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x$.

(b) Obtain the most general solution of the equation $y(x) = \lambda \int_0^{2\pi} \sin(x + \xi) \, y(\xi) \, d\xi + F(x)$ when $F(x) = x$ and and when $F(x) = 1$, under the assumption that $\lambda \neq \pm 1/\pi$.

(2 × 4 = 8 weightage)