D 53489 (Pages : 3) Name..................................

Reg. No..................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2014

(UG—CCSS)

Core Course

Mathematics

MM IB 01—FOUNDATIONS OF MATHEMATICS

(2010 Admission Onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Objective Type Questions)

Answer all **twelve** *questions.*
Each bunch of **four** *questions carries 1 weightage.*

1. Find x and y if $(y-2, 2x+1) = (x-1, y+2)$.
2. Find R^{-1} if $R = \{(1, y)\,(1, z), (3, y)\}$.
3. $A = \{1, 2, 3, 4\}$, R is a relation on A given by $R = \{(1,1)\,(2,2)\,(2,3)\,(3,2)\,(4,2)\,(4,4)\}$. Is R symmetric ?
4. What is the cardinality of the set $\{a, \{a\}, \{a, \{a\}\}\}$?
5. If $f : R \to R$, $g : R \to R$ be defined as $f(x) = x^2$, $g(x) = x + 4$ find $gof(x)$.
6. Fill in the blanks , $n! =$ ———— $(n-1)!$ if $n > 0$.
7. If $A = \{a, b\}$, $B = \{4, 5, 6\}$, find the number of functions from A into B.
8. Write the converse of the statement $p \to q$.
9. Write the truth table for $p \to q$.
10. What is the negation of the statement $\forall x\ p(x)$.
11. State the rule of inference 'Modus ponens'.
12. What is the truth value of the statement $\exists x\,(2x = 3x)$ if the domain consists of all integers.

(12 × ¼ = 3 weightage)

Turn over

Part B (Short Answer Type Questions)

Answer all **nine** *questions .*
Each question carries 1 weightage.

13. Find A × B and B × A if A = {1, 2}, B = { a, b}.

14. Let $A_i = \{1, 2, 3 \ldots . i\}$. Find $\bigcup_{i=1}^{n} A_i$.

15. Find floor function of (1) $\lfloor 7 \cdot 5 \rfloor$ (2) $\lfloor -2 \cdot 5 \rfloor$.

16. Let A = {1, 2, 3, 4} f and g are two functions on A defined by

$$f = \{(1,3)\,(2,1)\,(3,4)\,(4,3)\}$$

$$g = \{(1,2)\,(2,3)\,(3,1)\,(4,1)\}$$ find $f \circ g$.

17. What is the negation of the statement

'There is an honest politician'.

18. What is the truth value of $\exists x\, p(x)$ where $p(x): x^2 > 10$ and the domain consists of {1, 2, 3,

19. Define Tautology. Show $p \wedge \neg p$ is not a tautology.

20. Give a proof by contra position 'if n is an integer and $3n + 2$ is odd, then n is odd'.

21. Which rule of inference is used in the following argument :

"Alice is a Mathematics major. Therefore Alice is either a Mathematics major or a Computer Sc major".

(9 × 1 = 9 weigh

Part C (Short Essay Questions)

Answer any **five** *questions.*
Each question carries 2 weightages.

22. Show that the relation $a \equiv b \pmod{n}$ is an equivalence relation.

23. Find a formula for inverse of $g(x) = \dfrac{2x-3}{5x-7}$.

24. Let $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x - 3$ show f is one-to-one and onto.

25. Show $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

26. Express the mathematical statement using predicates, quantifiers and logical connectives.

    'The product of two negative real numbers is positive'.

27. Translate the following quantifications into English statement if the domain consists of all real numbers

    '$\exists x \, \forall y \, (xy = y)$'.

28. Construct a truth table for the compound proposition $(p \to q) \wedge (\neg p \to q)$.

(5 × 2 = 10 weightage)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Let $f : \mathrm{A} \to \mathrm{B}$, $g : \mathrm{B} \to \mathrm{C}$ be two functions prove :

    (a) If *gof* is one-to one then *f* is one-to-one.

    (b) If *gof* is onto then *g* is onto.

30. (a) Write the contra positive, inverse and converse of

    'If it is rainy, then the pool will be closed'.

    (b) Which rule of inference is used in the argument ?

    If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore if I go swimming, then I will sunburn'.

31. (a) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

    (b) Give a proof by contradiction to prove 'if n is an integer and $n^3 + 5$ is odd, then n is even.

(2 × 4 = 8 weightage)