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(Pages : 4)

Name.................................

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SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Complementary Course—Statistics

ST 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. The joint distribution function of (X, Y) is equivalent to the probability :

   (a) $p(X = x, Y = y)$.
   (b) $p(X \leq x, Y \leq y)$.
   (c) $p(X = x \; Y \leq y)$.
   (d) $p(X \leq x, Y = y)$.

2. If X and Y are independent r.vs. then :

   (a) $\text{Cov}(X, Y) = E(X) \cdot E(Y)$.
   (b) $\text{Cov}(X, Y) = E(XY)$.
   (c) $E(XY) = E(X) \cdot E(Y)$.
   (d) $E(XY) < E(X) \cdot E(Y)$.

3. $E(E\, X/Y) =$

   (a) $E(X)$.
   (b) $E(Y)$.
   (c) $E(X/Y)$.
   (d) $E(Y/X)$.

4. $\text{Cov}(c + X, d + Y) =$

   (a) $(c + d)\,\text{Cov}(X, Y)$.
   (b) $cd\,\text{Cov}(X, Y)$.
   (c) $c^2 d^2\,\text{Cov}(X, Y)$.
   (d) $\text{Cov}(X, Y)$.

5. If X follows uniform distribution in $[a, b]$ then $V(X) =$

   (a) $\frac{(b-a)^2}{2}$.
   (b) $\frac{(b-a)^2}{12}$.
   (c) $\frac{b+a}{2}$.
   (d) $\frac{(b+a)^2}{12}$.

Turn over

6. The distribution for which mean = variance is :

   (a) Poisson. (b) Binomial.

   (c) Uniform. (d) Exponential.

7. The mode of the geometric distribution $p(x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2, 3, \ldots$ is :

   (a) Zero. (b) One.

   (c) ½. (d) Does not exist.

8. If $\log_e X$ follows normal distribution, then probability distribution of X is :

   (a) Exponential. (b) Normal.

   (c) Rectangular. (d) Log normal.

9. If $X \sim N(0,1)$ then $p(1 \times 1 < 1)$ is :

   (a) 0.3413. (b) 0.6826.

   (c) 0.1587. (d) 0.3174.

10. If X denote the number appearing when a fair die is thrown, then the probability distrib of X is :

    (a) Binomial. (b) Geometric.

    (c) Uniform. (d) Poisson.

11. The points of inflexion of the normal curve are :

    (a) $\mu \pm \sigma$. (b) $\mu \pm 1$.

    (c) $\mu \pm 3\sigma$. (d) $\mu \pm 3$.

12. For a binomial distribution, mean = 4 and variance = $\frac{4}{3}$. Then $p(X = 0)$ is :

    (a) $\left(\frac{2}{3}\right)^6$. (b) $\left(\frac{1}{3}\right)^6$.

    (c) $6\left(\frac{1}{3}\right)^6$. (d) $6\left(\frac{2}{3}\right)^6$.

(12 × ¼ = 3 weigh

Part B (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries 1 weightage.

13. Define covariance of (X, Y) in terms of expectations.

14. Define Pareto distribution.

15. If $X \sim N(25, 3)$ and $Y \sim N(20, 4)$ and X and Y are independent, then find the distribution of $X + Y$.

16. Define conditional p.d.f of Y/X.

17. If X follows Beta distribution of type 1 with parameters m and n, what is its mean.

18. Define Cauchy distribution.

19. If $X \sim N(16, 2)$, find an upper bound for $p(|X - 16| > 6)$ using Chebychev's inequality.

20. Define convergence in probability.

21. What is the m.g.f. of $N(\mu, \sigma)$.

(9 × 1 = 9 weightage)

Part C (Short Essay Type Questions)

Answer any ***five.***
Each question carries 2 weightage.

22. X and Y are discrete r.vs. having joint p.d.f. $f(x, y) = \frac{2x + y}{27}$ $x = 0, 1, 2,\ y = 0, 1, 2.$

Examine whether X and Y are independent.

23. $f(x, y) = 8xy \quad 0 < x < y < 1$ find E (Y/X).
$\quad\quad\quad = 0 \quad$ elsewhere.

24. Show that for a Poisson distribution with unit mean, M.D. about mean is $\frac{2}{e}$.

25. Find the mean and variance of a Geometric distribution $f(x) = q^x \cdot p$; $x = 0, 1, 2, 3, \ldots.$

Turn over

26. State Lindeberg-Levy form of CLT.

27. Show that a linear combination of independent normal variates is also a normal variate.

28. Obtain the m.g.f. of an exponential distribution and hence find its mean and variance.

(5 × 2 = 10 weightage)

Part D (Long Essay Type Questions)

Answer any **two.**
Each question carries 4 weightage.

29. If μ_r is the r^{th} central moment of the binomial distribution B (n, p), prove that :

$$\mu_{r+1} = pq\left[\frac{d\mu r}{dp} + nr\,\mu_{r-1}\right].$$ Hence obtain μ_2 and μ_3.

30. (a) State important properties of normal distribution.

(b) Of a large group of students, 5% are under 150 cm and 40% are between 150 cm and 162 cm in height. Assuming normal distribution, find Mean and SD of height.

31. (a) State Weak law of large numbers.

(b) If X_i assumes values i and $-i$ with equal probabilities, show that the law of large numbers cannot be applied to the independent variables $X_1, X_2, X_3 \ldots$.

(2 × 4 = 8 weightage)