$\qquad$
$\qquad$

## FIRST SEMESTER B.C.A. DEGREE EXAMINATION, NOVEMBER 2019

(Supplementary/Improvement)
(CUCBCSS-UG)

## CC17U BCA1 C01 - MATHEMATICAL FOUNDATIONS FOR COMPUTER APPLICATIONS <br> (Mathematics - Complementary Course) <br> (2017-2018 Admissions) <br> Time: Three Hours <br> Maximum: 80 Marks

## PART A

Answer all questions. Each question carries 1 mark.

1. Define involutary matrix.
2. What is the rank of an $n \times n$ non-singular matrix?
3. Write the characteristic equation of $A=\left[\begin{array}{cc}8 & -4 \\ 2 & 2\end{array}\right]$.
4. Find the component and length of vector with initial point $P(6,2,-3)$ and terminal point $Q(-1,-1,2)$.
5. The system of linear equations $A X=B$ is $\ldots \ldots$. If $A$ and $[A B]$ have the same rank
6. Find $\frac{d y}{d x}$ if $y=\log \left(\sqrt{x^{2}+1}\right)$.
7. Evaluate $\lim _{x \rightarrow 1} \frac{x^{2}+4}{x+2}$
8. If $y=e^{x} \log x$, prove that $\frac{d y}{d x}=y+\frac{e^{x}}{x}$
9. Evaluate $\int \frac{\left(x^{2}+1\right)^{2}}{x^{3}} d x$
10. Evaluate $\int_{a}^{b} \frac{1}{x} d x$

## PART B

Answer all questions. Each question carries 2 marks.
11. Find all the values of $x \cdot y \cdot z$ and $a$ which satisfy the matrix equation

$$
\left[\begin{array}{ll}
x+3 & 2 y+x \\
z-1 & 4 a-6
\end{array}\right]=\left[\begin{array}{cc}
0 & -7 \\
3 & 2 a
\end{array}\right]
$$

12. Find a value of $\mu$ such that the vectors $\vec{a}$ and $\vec{b}$ are perpendicular where $\vec{a}=[2,3,4]$ and $\vec{b}=[3,2,-\mu]$
13. Find $\frac{d y}{d x}$ if $y=x^{x}$
14. The slope of a curve at $(x, y)$ is $9 x$. It passes through the origin. Show that its equation $9 x^{2}=2 y$
15. Show that $\int_{0}^{\pi / 2} \sin ^{2} x d x=\int_{0}^{\pi / 2} \cos ^{2} x d x$

## PART C

Answer any five questions. Each question carries 4 marks.
16. Find the rank of $A=\left[\begin{array}{cccc}1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7\end{array}\right]$
17. Show that the value of the determinant $D=\left|\begin{array}{ccc}x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14\end{array}\right|$ is independent of $x$ and prove that its value is -2 .
18. Differentiate from first principle $\sin x$
19. $(x+y)^{m+n}=x^{m} y^{n}$; find $\frac{d y}{d x}$
20. If $y=x^{4}-3 x^{3}+3 x^{2}+5 x+1$, prove that $\frac{d^{2} y}{d x^{2}}$ is negative, when $x$ lies between $\frac{1}{2}$ and 1
21. State chain rule of differentiation of composite functions. Using chain rule find $\frac{d y}{d x}$, where $y=9 u^{2}$ and $u=1-\frac{3}{2} x^{2}$
22. Evaluate $I=\int \frac{x^{2}}{(x+1)(x+2)^{2}} d x$
23. Evaluate $\int \log x d x$

## PART D

Answer any five questions. Each question carries 8 marks.
24. Find value of 'a' such that $x+y+z=3 ; x+2 y+2 z=6 ; x+a y+3 z=2$; have
(a) No solution
(b) Unique solution
25. Find the eigen values and eigen vector corresponding to any eigen value of the matrix $\left[\begin{array}{ccc}8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3\end{array}\right]$
26. (a) Find $\frac{d y}{d x}$ if $x=\frac{2 a t}{1+t^{2}}, y=\frac{1-t^{2}}{1+t^{2}} \quad$ (b) Find $\frac{d y}{d x}$ when $y=(1+2 x)^{x}$
27. (a) If $y=x^{3} \log \frac{1}{x}$, prove that $\frac{d^{2} y}{d x^{2}}-\frac{2}{x} \frac{d y}{d x}+3 x=0$
(b) If $y=x^{2} \log _{e} x^{2}$, find $\frac{d^{2} y}{d x^{2}}$, when $x=1$
28. (a) The slope of at any point $(x, y)$ of a curve is $\frac{x+1}{y+1}$. If the curve passes through the origin, find the equation of the curve.
(b) Evaluate $\int \frac{x^{2}+5 x+2}{(x+2)(x+3)} d x$
29. (a) Evaluate $\int x \log x \mathrm{dx}$
(b) Evaluate $\int \frac{\log x}{(1+\log x)^{2}} d x$
30. Evaluate $\int_{0}^{\pi / 2}(\sqrt{\sin \theta}) \cos ^{5} \theta d \theta$
31. Test for consistency and if consistent solve the system of equations

$$
2 x-y+z=7 ; 3 x+y-5 z=13 ; x+y+z=5
$$

