23U110 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC20U MTS1 B01 - BASIC LOGIC AND NUMBER THEORY

(Mathematics - Core Course)

(2020 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Write the truth table of conjunction.
2. Let $p \to q$. If $\triangle$ ABC is equilateral, then it is isosceles. Write the converse and inverse of the statement.
3. Rewrite the given propositions symbolically, where UD = set of real numbers.
   a) For each integer x, there exists an integer y such that $x + y = 0$.
   b) There are integers x and y such that $x + y = 5$.
4. Test the validity of the argument
   $p \leftrightarrow q$
   $\sim p \vee r$
   $\sim r$
   $\overline{\therefore \sim q}$
5. Prove directly that the sum of any two even integers is even.
6. State weak version of induction.
7. Compute the first four terms of the sequence defined recursively : $a_0 = 1, a_n = a_{n-1} + n$
8. Prove or disprove if p is prime, then $p^2 + 1$ is prime.
9. Find the five consecutive composite numbers less than 100.
10. Write a linear combination of 12, 15, and 21.
11. State Dirichlet's Theorem.
12. Give an example for diophantine equation.
13. Using divisibility test determine whether 398008 and 576 are divisible by 8.

14. Let p be a prime and a any integer such that p does not divide a. Then show that a^{p-2} is an inverse of a modulo p.

15. Define Euler's phi function and compute $\phi(21)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the number of positive integers less than or equal to 2000 and divisible by 3, 5, or 7.

17. Using the Euclidean Algorithm, find the gcd of 2076, 1076

18. Prove that if p be a prime and $p|a_1a_2....a_n$, where $a_1, a_2, ...a_n$ are positive integers, then $p|a_i$ for some i, where $1 \leq i \leq n$.

19. Find the positive factors of 90.

20. Let $a \equiv b(mod m)$ and $c \equiv d(mod m)$. Then show that $a + c \equiv b + d(mod m)$ and $ac \equiv bd(mod m)$.

21. Find the units digits in the decimal value of $1776^{1777^{1778}}$.

22. Solve the congruence $28a \equiv 119(mod 91)$.

23. Using inverses, find the incongruent solution of $4x \equiv 11(mod 13)$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (a) Verify $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.
    (b) Using the laws of logic simplify the boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$.

25. (a) Prove that if a and b be positive integers, then $[a, b] = \frac{ab}{(a,b)}$.
    (b) Compute $[252, 360]$

26. (a) If n is a positive integer such that $(n-1)! \equiv -1(mod n)$, then show that n is a prime.
    (b) Prove that a positive integer a is self-invertible modulo p if and only if $a \equiv \pm 1(mod\, p)$

27. (a) Using Euler's theorem find the remainder when 245^{1040} is divided by 18.
    (b) Solve the linear congruence $23x \equiv 17(mod\, 12)$.

(2 × 10 = 20 Marks)

****