CC19PPHY3C09:QUANTUM MECHANICS - II

A Part

1. Explain briefly the principle of time independent perturbation theory.
2. Using perturbation theory calculate the first order correction for ground state energy of the anharmonic oscillator having a potential energy U=$\frac{1}{2}m\omega^2x^2 + ax^3$ where a is a constant.
3. How time independent perturbation theory can be used to calculate eigen values.
4. Give the magnitude of the first order perturbation energy for a non-degenerate case.
5. Show that a perturbation removes degeneracy of a state.
6. What is meant by degeneracy and lifting of degeneracy?
7. What is Zeeman effect? How can we study it by perturbation theory?
8. How you will find the first order perturbation energy for a degenerate case?
9. Write a short note on hyperfine splitting in the ground state of hydrogen.
10. What is Zeeman effect?
11. Distinguish between Normal and Anomalous Zeeman effects?
12. Explain the linear stark effect in hydrogen atom
13. What do you mean by linear stark effect?
14. What is the effect of the application of an electric field in the linear Stark effect?
15. How do you calculate the energies of an excited state of a system using variational method?
16. Outline the theory of variational principle in approximation method.
17. Discuss the principles of variational method
18. Obtain Schrodinger equation from Ritz variation principle.
19. Discuss how we can get correct eigen value by Ritz variational principle.
20. Show that the variational method always gives an upper limit to the ground state energy of the system.
21. 1. Discuss the connection formula in WKB approximation.
22. What is the criterion for validity of WKB approximation?
23. Briefly explain the validity of WKB approximation.
24. What is a classical turning point?

25. Explain Bohr-Sommerfeld quantisation theory.
26. Write down the expression for Bohr-Sommerfeld quantization rule in the case of a bound system.
27. What is transmission coefficient?
28. Write and explain the connection formulae for a potential well with no vertical walls
29. In WKB approximation, why we need connection formula.
30. Write and explain the connection formulae for a potential well with one vertical wall
31. Write and explain the basic equation of first order time dependent perturbation theory
32. What do you mean by constant perturbation? Give an example.
33. Give the equation for transition probability from a state |i> to state |f>
34. State and explain Fermi's Golden rule for transition to continuum.
35. A system is subjected to a perturbation which lasts from time t=0 to $t=t_0$ and which is constant during this time. What is the transition probability?
36. Write an expression for transition probability when a constant perturbation is acting on the system, and explain.
37. Obtain the expression for total scattering cross section
38. What do you mean by scattering length?
39. Give the equation for differential scattering cross section in born approximation
40. Distinguish between stimulated and spontaneous emission.
41. Give the basic equation of the first order time-dependent perturbation theory and mention any two applications.
42. What is meant by constant perturbation? Describe the condition for the perturbation to induce a transition.
43. Why does spontaneous emission far exceeds stimulated emission in the visible region?
44. Outline the principles of dipole approximation.
45. Explain the principle of detailed balance.
46. Write a short note on scattering amplitude.
47. Explain the method of partial wave analysis.
48. What do you mean by scattering length?
49. What are turning points? Give its significance.

50. Show that the effect of the scattering potential is to shift the phase of each outgoing partial wave.
51. Briefly describe the effect of scattering potential on a partial wave
52. What change occurs to an incoming spherical wave when it is subjected to a central potential?
53. Explain Optical theorem in scattering.
54. what is the relevance of optical theorem in scattering?
55. Give the formula which gives the energy-dependence of the total (s-wave) cross-section at low energies
56. Obtain Weyl's equation for neutrinos.
57. Write the Weyl equation and explain.
58. What are the difficulties in the interpretation of K-G equation as a quantum mechanical equation?
59. What are the arguments used in deriving the Klein-Gordon equation?
60. Why we say about the helicity of neutrinos instead of its spin?
61. Bring out the difficulties in the probability interpretation of the Klein-Gordon wave equation.
62. What are the drawbacks of Klein-Gordon Equation?
63. What are the limitations of K.G equation?
64. What do you mean by negative energy states ?
65. Give any 4 properties of Dirac matrix.
66. How we can say that Dirac particles are spin half particles
67. Explain what is meant by Dirac Spin matrices.
68. Write down the explicit form of Dirac matrices.
69. What are Dirac matrices? Give any two its properties.
70. Explain what is meant by Paul Spin matrices.
71. Discuss the stability of Dirac vacuum.
72. Give two important properties of Dirac matrices.
73. Deduce the covariant form of Dirac equation.
74. Obtain probability density associated with the Dirac equation
75. Give the Pauli equation for electrons
76. Obtain equation of continuity from Dirac equation.
77. Explain why Schrodinger equation is not relativistically valid.
78. Briefly explain Hole theory of electron.

79. Justify the statement that Dirac equation has a sensible non-relativistic limit.

80. Schrodinger equation fails to give the correct wave equation for relativistic particles. Why?

B Part

81. Derive the first-order correction to the energy for a non-degenerate system using time-independent perturbation theory.

82. Explain the second-order correction to energy in non-degenerate perturbation theory with an example.

83. Discuss in detail, the degenerate perturbation theory by assuming the two-fold degeneracy. Explain how it can be generalized to higher order degeneracy.

84. Solve the problem of the anharmonic oscillator using perturbation theory.

85. Discuss the degenerate perturbation theory and solve for two-fold degeneracy in a quantum system.

86. Illustrate the relativistic corrections in the hydrogen atom, focusing on fine structure and spin-orbit coupling.

87. Compare the weak-field, strong-field, and intermediate-field Zeeman effects with diagrams.

88. Analyze the linear Stark effect in the hydrogen atom.

89. Applying time independent perturbation theory, account for stark splitting in the first excited state of Hydrogen atom

90. How time Independent perturbation theory can be used to explain Stark effect. The levels undergoing splitting in Stark effect doesn't undergo splitting in Zeeman effect. Comment

91. (i). Briefly explain the variational method used for obtaining approximate value of ground state energy of a system. (ii). Obtain the ground state energy for Helium atom using variational method.

92. 1. Discuss the theory of WKB approximation. Use it to study the problem of barrier tunneling.

93. Discuss variation method for the evaluation of eigen values. Obtain the ground state energy of Helium atom by variation method.

94. Use the variational method to estimate the ground state energy of the Helium atom.

95. Use the variational method to estimate the energies of a one dimensional harmonic oscillator in the ground state and first excited state.

96. Apply the Ritz variational method to the helium atom and compare it with experimental results.

97. Briefly explain the theory of WKB approximation. Using WKB approximation obtain the expression for transmission coefficient of a potential barrier.

98. Derive the WKB wavefunction in the classical region for a potential well with two vertical walls.

99. Prove that the WKB approximation gives correct energy Eigen values of all the states of a harmonic oscillator.

100. Derive the Bohr-Sommerfeld quantum condition from WKB method. Using this calculate the energy eigen values of all the states of harmonic oscillator.

101. Use the WKB approximation to analyze tunneling and derive the connection formulae.

102. Use WKB method to calculate transmission and reflection coefficient for a particle penetrating through an arbitrary potential V (x)

103. What are connection formulae in WKB approximation method? Derive connection formulae and apply it to a potential well with one vertical wall.

104. Discuss the optical theorem and its application in quantum scattering problems.

105. Show that a hydrogen atom in its first excited state behaves as though it has permanent electric dipole moment that can be oriented in three different ways.

106. Explain the method of calculating transition probability using time dependent perturbation theory. Derive an expression for transition probability, when a system is subjected to constant perturbation.

107. Obtain the expression for total transition probability for unit time when an atom interact with an electromagnetic field.

108. Obtain the expression for Fermi's Golden rule.

109. Derive Fermi's Golden Rule and discuss its applications in quantum transitions.

110. Solve the scattering cross section in the Born approximation for a given potential.

111. Explain the harmonic perturbation and its role in radiative transitions in atoms.

112. Explain the method of calculating transition probability using time dependent perturbation theory. Derive an expression for transition probability, when a system is subjected to harmonic perturbation.

113. Apply the time dependent perturbation theory to discuss the radiative transitions in atoms.

114. Using the method of partial wave analysis, explain scattering by square well potential.

115. Define scattering amplitude and scattering cross section. How they are related? Using partial wave analysis derive the expression for scattering cross section

116. Compare the scattering cross section in the Born approximation with that obtained by the method of partial waves.

117. Derive the method of partial waves for scattering by a central potential.

118. Discuss the optical theorem and its application in quantum scattering problems.

119. Solve the Klein-Gordon equation for a free particle and interpret the solution

120. Obtain Klein-Gordon equation. Discuss how the reinterpretation helped to overcome the limitations

121. Explain the Klein-Gordon equation and its limitations for spin-1/2 particles.

122. Obtain the free particle solutions of Dirac relativistic equation. Discuss the negative energy states.

123. Starting from Dirac Hamiltonian obtain the free particle solution of Dirac Equation.

124. Discuss the properties of Dirac matrices and their role in relativistic quantum mechanics.

125. Derive the plane wave solutions of Dirac equation. Write the equation for a Central field.

126. Obtain the covariant form of the Dirac equation. What are Dirac matrices? Write these matrices and discuss in detail about the properties of Dirac matrices.

127. From Dirac equation obtain Pauli's equation for electron. Explain spin orbit interaction.

128. Derive the Dirac equation for a free particle and discuss the concept of spinors.

129. Analyze the non-relativistic limit of the Dirac equation for a particle in an external magnetic field.

130. Derive the equation of continuity for a Dirac particle and explain its physical significance.

131. Explain the significance of hole theory in the Dirac equation and its implications in quantum mechanics.

C Part

132. A rigid rotator in a plane is acted on by a perturbation represented by $H' = \frac{V_0}{2}(3cos^2\emptyset - 1)$, V_0=constant. Calculate the ground state energy up to the second order in the perturbation

133. State and explain the first-order correction to the wavefunction for a non-degenerate system.

134. A perturbation in the form H'=ax is applied to a particle under potential V(x)=0, for $0 \le x \le \pi$;V(x)= ? otherwise. Calculate the correction in ground state energy.

135. Calculate the first order correction to the eigen values of a quartic oscillator.

136. A simple harmonic oscillator of mass m_0 and angular frequency ω is perturbed by an additional potential Cx^4. Evaluate the second order correction to the ground state energy of the oscillator.

137. Obtain the second-order perturbation theory formula for non-degenerate states.

138. A one-dimensional box of width L contains two spinless particles each of mass m. The interaction between the particles is described by a potential $V(x_1,x_2)=a\delta(x_1-x_2)$. Find the first-order correction in energy.

139. The ground state wave function for 1D harmonic oscillator $\psi_0 = Ae^{-\frac{\alpha^2x^2}{2}}$ is perturbed by a potential $E_0\left(\frac{\alpha x}{10}\right)^4$, Find the first order change in ground state energy.

140. A simple harmonic oscillator is perturbed by a harmonic potential so that the result Hamiltonian is given by $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda x^2$ Calculate the first order perturbation energy.

141. We have a Hamiltonian H= $\begin{pmatrix} 1+\propto & \propto \\ \propto & 1+\propto \end{pmatrix}$. Calculate the first order correction in energy

142. The Hamiltonian H_0 for a 3 state quantum system is given by $H_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ when perturbed by H'=ε $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ where ε<<1, find the resulting shift in the energy eigen value $E_0=2$

143. Explain the concept of two-fold degeneracy in quantum mechanics.

144. Consider a quantum system with Hamiltonian $H=V_0 \begin{pmatrix} (1-\epsilon) & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$ where V_0 is a constant and Ɛ is some small number(Ɛ<<1). Use degenerate perturbation theory to find the first order correction to the two initially degenerate eigen values.

145. What is the significance of the relativistic correction to the hydrogen atom?

146. Discuss the anharmonic oscillator as an example of perturbation theory.

147. Explain the concept of spin-orbit coupling in the context of hydrogen atom spectra.

148. What is the fine structure of hydrogen and how is it corrected relativistically?

149. Apply time independent perturbation theory to explain weak-field Zeeman effect.

150. Define and differentiate between the weak-field and strong-field Zeeman effects.

151. Discuss the hyperfine splitting in atomic spectra

152. Applying degenerate perturbation theory, calculate the energy levels of the n = 2 state of a Hydrogen atom placed in an external uniform electric field along the positive z-axis.

153. For a particle of mass m moving in the potential V(x) = kx for x>0 and V(x) = ∞ for x<0, where k is a constant. Estimate the ground state energy of the system by optimizing the trial wave function xe−αx

154. If A and B are operators whose components commute with α; show that (α.A)(α.B) = (A.B) +i σ^D (AxB) where $\sigma^D = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$

155. Use variational method to find the ground state energy of one dimensional harmonic oscillator using the trial wave function Ψ= A $e^{-\alpha x^2}$

156. Apply the variational principle to find the ground state energy for one dimensional harmonic oscillator.

157. Evaluate, by the variation method, the energy of the first excited state of a linear harmonic oscillator using the trial function $\phi = Nx \exp(-\lambda x^2)$

158. 1. Discuss the “good quantum numbers” in weak-field Zeeman splitting and obtain the energy of the n^{the} state under weak-field Zeeman splitting.

159. Discuss the variational method and its application to the harmonic oscillator.

160. Use the variational method to find the ground state energy of Helium atom.

161. The hamiltonian of a system is given by $H = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} - a\delta(x)$ where a is a constant and $\delta(x)$ is Dirac's delta function.Estimate the ground state energy of the system using a Guassian trial function

162. Optimize the trial wave function $e^{-\alpha r}$ and hence obtain the ground state energy of the Hydrogen atom.

163. Obtain energy levels of a particle moving under the potential V (x) = k |x| by WKB method.

164. obtain the energy values of harmonic oscillator by the WKB method

165. Determine in the WKB approximation, the energy levels of a particle moving in a uniform gravitational field when the motion is limited from below by a perfectly reflecting plane.

166. Use the WKB approximation to calculate the energy levels of a spin less particle of mass m moving in a one dimensional box with walls at x=0 and x=-L

167. Explain tunnelling in quantum mechanics using the WKB approximation.

168. Derive the Bohr-Somerfield quantum condition using WKB method.

169. Find the energy levels of a particle in a potential V (x) = Vo|x|,Vo being positive constant using Bohr-Somerfield quantization rule.

170. Write the connection formulae for WKB approximation.

171. Using WKB method solve the one dimensional potential well given by $V(x) = 0$ for $-a<x<a$, $V(x)=\infty$ for $x>a$

172. Derive the WKB wavefunction for a potential well with no vertical walls.

173. A system in an unperturbed state “n” is suddenly subjected to a constant perturbation H(r). Find the transition probability from the initial state “n” to the final state “k”.

174. Derive the first-order time-dependent perturbation theory formula for constant perturbation.

175. Find out the transition probability for a perturbing potential which has no explicit dependence on time in time dependent perturbation theory.

176. What is Fermi’s Golden Rule and how does it apply to transitions to a continuum

177. Explain the Born approximation for scattering and its limitations.

178. Explain the concept of radiative transitions in atoms using harmonic perturbation.

179. Discuss the transititon probability of a system under harmonic perturbation

180. The differential cross section of scattering by a target is given by $\frac{d\sigma(\theta,\varphi)}{d\Omega} = a^2 + b^2 cos^2\theta$. If N is the flux of incoming particle ,find the number of particles scattered per unit time.

181. What is the scattering amplitude and how is it calculated?

182. Consider the partial wave analysis of scattering by a potential V(r) and derive an expression for the phase shift δ_l in terms of V(r) and the energy E of the incident wave.

183. By using partial wave analysis, obtain the expression for scattering cross section when the size of the scattering center is greater than the wave length of the incident particle. Compare the result with classical scattering cross section

184. Calculate the total scattering cross section for a low energy particle from a potential given by V= -Vo for r < a and V = 0 for r > a.

185. Obtain the relation between scattering cross section and scattering amplitude

186. The differential scattering cross section in a certain case is given to be $\sigma(\theta)=\alpha+\beta\cos\theta+\gamma\cos^2\theta$
(a)What is the scattering amplitude? (b) Deduce the total scattering cross-section and show that it is consistent with the optical theorem.

187. Show that for zero energy scattering the total scattering cross section is given by $\sigma = 4\pi a^2$.

188. Prove that the effect of the central scattering potential is to shift the phase of each outgoing partial wave.

189. Define and explain the scattering cross section for a central potential.

190. Briefly explain the optical theorem and explain its significance.

191. Discuss the scattering of particles by a square-well potential.

192. Obtain the Hamiltonian form of Klein-Gordon equation.

193. State and explain the Klein-Gordon equation for relativistic particles

194. Derive the equation of continuity using K.G equation.

195. Starting from K.G equation, derive the equation of continuity.

196. Derive expressions for the probability density and probability current density in Dirac theory.

197. Discuss the non-relativistic limit of the Dirac equation.

198. Show that angular momentum associated with orbital motion of Dirac particle is not a constant of motion.

199. Explain the properties of Dirac matrices.

200. Prove that the probability density associated with the Dirac equation is positive definite.

201. Show that $\{\gamma\mu, \gamma\nu\} = 2\delta\mu\nu$. Where γ are Dirac matrices.

202. What are the properties of Dirac spinors?

203. Show that the matrix $\sigma' = \begin{bmatrix} \sigma & 0 \\ \sigma & 0 \end{bmatrix}$ is not a constant of motion.

204. Consider a Dirac particle in an electromagnetic field and obtain the Pauli equation for an electron and show that the Dirac particles (positive energy ones) are electrons.

205. What is hole theory and how does it relate to the Dirac equation?