24U107 (Pages: 2) Name : ....

Reg. No : ....

FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2024

(FYUGP)

CC24U MAT1 CJ101 - DIFFERENTIAL CALCULUS

(B.Sc. Mathematics - Major Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer **all** questions. Each question carries 3 marks.

1. Find the domain of the functions $f(x) = 1 + x^2 + 2x^3$ and $g(x) = \dfrac{1}{1 + \sqrt{x}}$. [Level:2] [CO1]

2. Find $\lim\limits_{x \to -2^+} \dfrac{x^2 - 1}{2x + 4}$ and $\lim\limits_{x \to -2^-} \dfrac{x^2 - 1}{2x + 4}$. [Level:2] [CO1]

3. If $2 - x^2 \le g(x) \le 2 + \sin x$, find $\lim\limits_{x \to 0} g(x)$. [Level:2] [CO1]

4. Use chain rule to differentiate $y = 9x^4 + 6x^2 + 1$. [Level:3] [CO3]

5. Define instantaneous velocity and acceleration of an object moving along a coordinate line. [Level:1] [CO2]

6. Define continuity of a function at an interior point $x = c$ and name different types of discontinuities. [Level:1] [CO2]

7. Differentiate the function $f(x) = x + \frac{9}{x}$ and use it to find the slope of the tangent line at $x = -3$. [Level:3] [CO2]

8. Explain how the first derivative determines where a function is increasing or decreasing. [Level:1] [CO4]

9. Define vertical asymptote of the graph of a function. Find the vertical asymptote of $y = \sec x$. [Level:2] [CO5]

10. Explain the second derivative test for local extreme values. [Level:1] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer **all** questions. Each question carries 6 marks.

11. a) Apply the properties of limits to evaluate $\lim\limits_{t \to 1} \dfrac{t^2 + t - 2}{t^2 - 1}$. [Level:2] [CO1]

b) Apply the properties of limits to evaluate $\lim\limits_{v \to 2} \dfrac{v^3 - 8}{v^4 - 16}$.

12. Find the center and radius of the circles [Level:2] [CO1]

$$x^2 + y^2 + 4x - 6y - 3 = 0 \ \ and \ \ x^2 + y^2 + 4x - 4y + 4 = 0.$$

13. State the quotient rule and apply it to find the derivative of $y = \frac{t^2-1}{t^2+1}$. [Level:3] [CO2]

14. What is implicit differentiation? Use implicit differentiation to find the slope of circle [Level:3] [CO3]
$x^2 + y^2 = 25$ at the point $(3, -4)$.

15. Given $f'(x) = (x-1)^2 (x+2)^2$. What are the critical points of f? On what intervals is f [Level:3] [CO4]
increasing or decreasing?. At what points, if any, does f assume local maximum and
minimum values?

16. Apply the Mean Value Theorem to the function $f(x) = \sqrt{x-1}$ in the interval $[1, 3]$ and [Level:3] [CO4]
determine the value or values of c that satisfy the theorem.

17. Identify the inflection points of the function $y = x^4 - 8x^2 + 12$ and also identify the [Level:3] [CO5]
intervals on which the functions are concave up and concave down.

18. Find $\lim\limits_{x \to \infty} \left(\frac{\sin 2x}{x} \right)$ and $\lim\limits_{x \to \infty} \left(\frac{3 - (2/x)}{4 + (\sqrt{2}/x^2)} \right)$ [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

19. (a) Compute the absolute extrema of $h(x) = -3x^{2/3}, \ \ -1 \le x \le 1$. [Level:3] [CO4]
(b) Compute the absolute extrema of $g(x) = \sqrt{4 - x^2}, \ \ -2 \le x \le 1$.

20. Graph the rational function $y = \frac{x^3+1}{x}$. Include the graphs and equations of the asymptotes [Level:3] [CO5]
and dominant terms.

(1 × 10 = 10 Marks)

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