24U111 (Pages: 2) Name : ....

Reg. No : ....

FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2024

(FYUGP)

CC24U MAT1 CJ102 - ELEMENTARY NUMBER THEORY

(B.Sc. Mathematics - Major Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Show that every odd integer is of the form $6k+1, 6k+3$ or $6k+5$. [Level:2] [CO1]
2. Find the $gcd(72, 140)$. [Level:2] [CO1]
3. Define Diophantine Equation. Give an example. [Level:2] [CO2]
4. Define prime and composite numbers. Give examples. [Level:2] [CO2]
5. Prove that for arbitrary integers a and b, $a \equiv b \pmod{n}$ if a and b leave the same non negative remainder when divided by n. [Level:2] [CO3]
6. Prove that $a \equiv a \pmod{n}$ for arbitrary integer a. [Level:2] [CO3]
7. State Wilson's theorem. [Level:1] [CO4]
8. State Euler's theorem. [Level:2] [CO5]
9. Given integers a, b, c, show that $gcd(a, bc) = 1$ iff $gcd(a, b) = 1$ and $gcd(a, c) = 1$. [Level:2] [CO5]
10. List out some properties of Euler phi function. [Level:2] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. If n is odd, prove that $32|(a^2+3)(a^2+7)$. [Level:2] [CO1]
12. State and prove Euclid's lemma. [Level:2] [CO1]
13. Show that the quadratic equation $x^2+1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution iff $p \equiv 1 \pmod{4}$. [Level:1] [CO4]
14. Prove that $13^{\#}+1$ is not prime where $p^{\#}$ is the product of all primes that are less than or equal to p. [Level:2] [CO2]

15. Determine all solutions in integers of the Diophantine equation [Level:3] [CO2]

$1776x + 1976y = 4152$.

16. Find the solution of thesystem: [Level:3] [CO3]

$3x + 4y \equiv 5 \pmod{13}$

$2x + 5y \equiv 7 \pmod{13}$

17. Let p be a prime and suppose that p does not divides a. Then show that $a^{p-1} \equiv 1 \pmod{p}$. [Level:2] [CO3]

18. (a) Derive a formula to evaluate $\phi(n)$, for $n > 1$ [Level:2] [CO5]

(b) Using the formula evaluate $\phi(360)$.

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. Prove that every positive integer $n > 1$ can be expressed as a product of primes; this representation is unique, apart from the order in which the factors occur. [Level:2] [CO2]

20. (a) Define a linear congruence [Level:3] [CO3]

(b) Solve the linear congrunce $18x \equiv 30 \pmod{42}$

(c) Solve the linear congruence $6x \equiv 15 \pmod{21}$

(1 × 10 = 10 Marks)

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