

FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2024

(FYUGP)

CC24U MAT1 MN102 - CALCULUS OF SINGLE VARIABLE

(B.Sc. Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours

Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)Answer *all* questions. Each question carries 3 marks.

1. Compute $\lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$ [Level:3] [CO1]
2. Evaluate $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{3x - 12}$ [Level:3] [CO1]
3. Find $\lim_{x \rightarrow +\infty} \frac{5x^2 - 4x}{2x^2 + 3}$. [Level:3] [CO1]
4. Find $\frac{dy}{dx}$ if $y = x \sin x$. [Level:2] [CO2]
5. Find $\frac{dy}{dx}$ if $y = \cos(x^3)$. [Level:2] [CO2]
6. Using definition of derivative find $f'(x)$ if $f(x) = x^3 - x$. [Level:3] [CO2]
7. Evaluate $\lim_{x \rightarrow 0^+} x \ln x$. [Level:2] [CO4]
8. Find $\frac{dy}{dx}$ if $y = \sin^{-1} x + \cos^{-1} x$. [Level:2] [CO3]
9. Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing. [Level:3] [CO5]
10. Locate the critical points of $f(x) = 4x^4 - 16x^2 + 17$ and identify which critical points are stationary points [Level:2] [CO5]

(Ceiling: 24 Marks)**Part B** (Paragraph questions/Problem)Answer *all* questions. Each question carries 6 marks.

11. Let $f(t) = \begin{cases} t + 2, & t < 0 \\ t^3, & 0 \leq t \leq 2 \\ 4t & t > 2 \end{cases}$. Find the limits that exist. If the limit does not exist, explain why. (a) $\lim_{t \rightarrow 0} f(t)$ (b) $\lim_{t \rightarrow 1} f(t)$ (c) $\lim_{t \rightarrow 2} f(t)$ [Level:3] [CO1]

12. Find a value of the constant k , if possible, that will make the functions continuous everywhere. [Level:3] [CO1]
- $$(a) f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ \frac{k}{x^2}, & x < -3 \end{cases}$$
- $$(b) g(x) = \begin{cases} 9 - x^2, & x \geq 0 \\ \frac{k}{x^2}, & x < 0 \end{cases}$$
13. Discuss the continuity of the functions $f(x) = \sqrt{9 - x^2}$ and $g(x) = |x|$ [Level:3] [CO1]
14. Find an equation for the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$. [Level:2] [CO2]
15. State the product rule and use it to find $\frac{dy}{dx}$ if $y = (4x^2 - 1)(7x^3 + x)$. [Level:3] [CO2]
16. Use L'Hospital's Rule to evaluate (a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 2x - 8}$ (b) $\lim_{x \rightarrow +\infty} \frac{2x - 5}{3x + 7}$. [Level:3] [CO4]
17. Use logarithmic differentiation to find the derivative of $y = x\sqrt[3]{1 + x^2}$. [Level:3] [CO3]
18. Determine the intervals on which $f(x) = 7 + 24x - x^3$ is concave up and concave down. Locate all the inflection points, if any. [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any **one** question. The question carries 10 marks.

19. (a) Use implicit differentiation to find $\frac{dy}{dx}$ if $xy = 1$. [Level:3] [CO3]
 (b) Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if $x^3 + y^3 = 1$.
20. Sketch the graph of the equation $f(x) = 1 + 8x - x^2$ and identify the locations of the intercepts, relative extrema, and inflection points. [Level:3] [CO5]

(1 × 10 = 10 Marks)
