24U108 (Pages: 2) Name : ....

Reg. No : ....

FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2024

(FYUGP)

CC24U MAT1 MN105 - MATRIX THEORY

(B.Sc. Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Mark

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Solve the matrix equation for a, b, c and d. $\begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} 4 & d-2c \\ d+2c & -2 \end{bmatrix}$ [Level:2] [CO1]

2. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \\ -6 & 0 \\ -3 & 1 \end{bmatrix}$ Find $2A^T + B$ [Level:2] [CO1]

3. Solve the system of equations $x + y = 4$
$3x + 3y = 6$ [Level:2] [CO1]

4. If $p(x) = x^3 - 2x + 1$, compute p(A) for the matrix $A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$ [Level:3] [CO2]

5. Verify that AA^T is symmetric where $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ [Level:2] [CO2]

6. Verify that $(A^T)^{-1} = (A^{-1})^T$ where $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ [Level:2] [CO2]

7. Evaluate the determinant of the matrix $\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$. [Level:2] [CO4]

8. Find a terminal point Q of a nonzero vector $\bar{u} = \overrightarrow{PQ}$ with initial point $P(-1, 3, -5)$ and such that $\bar{u}$ has the same direction as $\bar{v} = (6, 7, -3)$. [Level:3] [CO5]

9. Find the norm of the vector $\bar{v} = (-3, 2, 1)$. Also find the unit vector in the direction of $\bar{v}$. [Level:3] [CO5]

10. Find the vector component of $\bar{u} = (3, 0, 4)$ along $\bar{a} = (2, 3, 3)$ [Level:3] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Using elementary row operations solve the system. [Level:2] [CO1, CO4]
$2x + 5y = 1$
$3x + 2y = 7$

12. Use the inversion algorithm to find A^{-1} if it exists. [Level:3] [CO2]

$A = \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}$

13. Verify that $(AB)C = A(BC)$ for the matrices [Level:2] [CO2]

$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 2 \\ 0 & 1 & -2 \end{bmatrix} B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \\ 0 & 2 \end{bmatrix} C = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

14. solve the system by inverting the coefficient matrix. [Level:2] [CO2]

$5x_1 + 3x_2 + 2x_3 = 4$
$3x_1 + 3x_2 + 2x_3 = 2$
$x_2 + x_3 = 5$

15. Given $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Show that $det(AB) = det(A)det(B)$. Also determine whether $det(A + B) = det(A) + det(B)$ [Level:3] [CO4]

16. Given A is a 3×3 matrix with $det(A) = 2$. Evaluate [Level:3] [CO4]

(a) $det(-A)$

(b) $det(A^{-1})$

(c) $det(2A^T)$

(d) $det(A^3)$

17. Find vector and parametric equations of the plane that passes through the point $P_0(-1, 1, 4)$ and is parallel to the vectors $\bar{v}_1 = (6, -1, 0)$ and $\bar{v}_2 = (-1, 3, 1)$ [Level:3] [CO5]

18. Find the area of the triangle determined by the points $P_1(1, -1, 2), P_2(0, 3, 4)$ and $P_3(6, 1, 8)$. [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. Solve by Gauss-Jordan elimination. [Level:2] [CO1]

$x_1 + x_2 + 2x_3 = 8$
$-x_1 - 2x_2 + 3x_3 = 1$
$3x_1 - 7x_2 + 4x_3 = 10$

20. Find adjoint of the matrix $\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$ Determine whether the matrix is invertible, and if so, use the adjoint method to find its inverse. [Level:3] [CO3]

(1 × 10 = 10 Marks)

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