FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - UG)

CC19U MTS1 C01 / CC20U MTS1 C01 - MATHEMATICS - I

(Mathematics - Complementary Course)

(2019 to 2023 Admissions - Supplementary/Improvement)

Time : 2.00 Hours   Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Find $\lim_{\Delta x \to 0} \dfrac{(\Delta x)^2 + 2\Delta x}{(\Delta x)^2 + \Delta x}$.

2. Differentiate $4x^9 - 6x^5 + 3x$ with respect to x.

3. Find the derivative of $h(x) = \dfrac{2x + 1}{x^2 - 2}$

4. If $y = (x^2 + 1)^{27}(x^4 + 3x + 1)^8$ Find $\dfrac{dy}{dx}$

5. If $x^3 + y^3 = xy$, compute $\dfrac{dx}{dy}$ in terms of x and y.

6. Find $\int \left(\dfrac{t^3 + t + 1}{t^5} \right) dt$

7. Show that the function $f(x) = \dfrac{3x^2}{x^3 - 2}$ is continuous at $x_0 = 1$.

8. The temperature at time t is given by $f(t) = \dfrac{(t + 1)}{t - 1}$ for $t < 1$. Is it getting warmer or colder at $t = 0$.

9. Use the second derivative test to find the local maximum and local minimum of the function $f(x) = x^3 - 6x - 3$

10. State Mean Value Theorem.

11. Find the sum $\sum_{i=3}^{99} i$

12. Using the fundamental theorem of calculus, Compute $\int_a^b x^2 dx$.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer *all* questions. Each question carries 5 marks.

13. Use the formal definition find the derivative of x^3.

14. Find the equation of the line tangent to the graph of $f(x) = \left[\dfrac{1}{x} - 2x \right] (x^2 + 2)$ at $x = \dfrac{1}{2}$.

15. Find the equation of the line tangent to the parametric curve $x = t^2 + 1, y = \dfrac{1}{t^4 + 1}$ at $t = 2$.

16. Find the critical points, endpoints, maximum and minimum points and values of the function $f(x) = \dfrac{1}{(1 + x^2)}$ on the intreval $[-2, 2]$

17. Evaluate $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$

18. (a) Check the integral $\int x(1 + x)^6 \, dt = \dfrac{1}{56}(7x - 1)(1 + x)^7 + C$

    (b) Evaluate $\int_0^2 x(1 + x)^6 \, dt$.

19. A parabolic doorway with base 6 feet and height 8 feet is cut out of a wall. How many square feet of wall space are removed?

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

20. (a) A bagel factory produces $30x - 2x^2 - 2$ dollars worth of bagels for each x worker hours of labor. Find the marginal productivity when 5 worker hours are employed.

    (b) Compute the second derivative $\dfrac{d^2}{dx^2}\left(\dfrac{x^3 - 1}{x^4 + 8}\right)$

21. (a) Find the volume of the sphere of radius r by using slice method.

    (b) Find the average value of $(x^2 \sin(x^3))$ on $[0, \pi]$.

(1 × 10 = 10 Marks)

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