

23U301

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Name:

Reg.No:

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(CBCSS - UG)
(Regular/Supplementary/Improvement)
CC19U MTS3 B03 / CC20U MTS3 B03 - CALCULUS OF SINGLE VARIABLE - II
(Mathematics - Core Course)
(2019 Admission onwards)

Time : 2.5 Hours

Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Define the natural exponential function $f(x) = e^x$. What are its domain and range?
2. Find the derivative of $f(x) = 2^x$.
3. Find the derivative of $g(x) = \tanh(1 - 3x)$.
4. Evaluate $\lim_{x \rightarrow 0} \frac{x^3}{x - \tan x}$
5. State comparison test for improper integrals.
6. Define bounded sequence.
7. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n-1}$ converges or diverges.
8. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ is absolutely convergent.
9. Find the Taylor series of $f(x) = e^{-3x}$ at $x = 0$
10. Describe the curves represented by the parametric equations $x = a \cos \theta$ and $y = a \sin \theta$, $a > 0$ with parameter interval $[0, 2\pi]$.
11. Find the slope of tangent line to the curve with the polar equation $r = \theta$ at $\theta = \pi$.
12. Find parametric equations for the line passing through the point $(-2, 1, 3)$ and parallel to the vector $\mathbf{v} = \langle 1, 2, -2 \rangle$.
13. Find an equation in rectangular coordinates for the surface with spherical equation $\rho \sin \phi = 3$.
14. Find the domain of $\bar{\gamma}(t) = \ln t\bar{i} + \cosh t\bar{j} + \tanh t\bar{k}$
15. The position vector of a moving object in a plane is given by $\bar{\gamma}(t) = t^2\bar{i} + t\bar{j}$; $t \geq 0$, find velocity and acceleration at $t = 2$

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer **all** questions. Each question carries 5 marks.

16. Find the inverse of f , where $f(x) = 3x - 2$. Then sketch the graph of f and f^{-1} on the same set of axes.
17. Find the derivative of $y = \sin^{-1}\left(\frac{\sin x}{1+\cos x}\right)$.
18. Determine whether the series $\sum_{n=1}^{\infty} \frac{2^n}{5^n}$ converges or diverges. If it converges find its sum.
19. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{3^n}$ is absolutely convergent, conditionally convergent or divergent.
20. Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n n! x^n}{2^n}$.
21. Find an equation of the tangent line to the curve $x = 2t - 1$, $y = t^3 - t^2$, at the point where $t = 1$.
22. Sketch the surface represented by the equation $x^2 + y^2 = z$.
23. Prove that $\frac{d}{dt}(\bar{u}(t) \cdot \bar{v}(t)) = \bar{u}'(t) \cdot \bar{v}(t) + \bar{u}(t) \cdot \bar{v}'(t)$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any **two** questions. Each question carries 10 marks.

24. Evaluate $\int \frac{\sec^2 3x}{4 - \tan 3x} dx$.
25. a) Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n-1}}$ converges or diverges.
b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.
26. (i) Find all the points of intersection of the cardioid $r = 1 + \cos\theta$ and the circle $r = 3\cos\theta$.
(ii) Find the area of the region that lies outside the cardioid $r = 1 + \cos\theta$ and inside the curve $r = 3\cos\theta$
27. a) Find the curvature of a circle of radius a .
b) Find the curvature of the curve $y = e^{-x^2}$.

(2 × 10 = 20 Marks)
