23U302 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 C03 / CC20U MTS3 C03 - MATHEMATICS - III

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Given $\mathbf{r}(t) = \frac{\sin 2t}{t}\mathbf{i} + (t-2)^5\mathbf{j} + t\ln t\mathbf{k}$. Find $\lim_{t\to 0^+} \mathbf{r}(t)$.
2. If $w = xy\ln(xz)$, find $\dfrac{\partial w}{\partial x}$ and $\dfrac{\partial w}{\partial z}$
3. Find the level surface of $f(x,y,z) = y+z$ passing through the point $(3,1,1)$
4. Show that the line integral $\displaystyle\int_{(1,1)}^{(2,4)} 2xydx + x^2dy$ is path independent.
5. State Green's theorem in the plane.
6. What do you meant by an orientable surface?
7. Define $Jacobian$ of a transformation.
8. Express the complex number $(2+3i)^2$ in the form $a+ib$.
9. Define the analyticity of a function at a point.
10. Find all values of z such that $e^z = \sqrt{3}+i$
11. Evaluate $\displaystyle\oint_C \frac{z}{2z+3}dz$, where C is the unit circle $|z| = 1$.
12. Evaluate $\displaystyle\int_{\pi i}^{2\pi i} \cosh z\, dz$

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Find the directional derivative of $f(x,y,z) = \frac{x^2-y^2}{z^2}$ at the point $(2,4,-1)$ in the direction of $\vec{i} - 2\vec{j} + \vec{k}$.
14. Find the curl and divergence of the vector field $\mathbf{F}(x,y,z) = yz\ln x\mathbf{i} + (2x-3yz)\mathbf{j} + xy^2z^3\mathbf{k}$
15. Find the volume of the solid bounded by the graphs of $2x+y+z=6, x=0, y=0, z=0$ in the first octant.

16. Convert the point $\left(4, \frac{7\pi}{4}, 0\right)$ given in cylindrical coordinates to rectangular coordinates.

17. If $\mathbf{F} = xy\mathbf{i} + y^2z\mathbf{j} + z^3\mathbf{k}$, evaluate $\iint_S \mathbf{F}.\mathbf{n}dS$ where S is the unit cube defined by $0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1.$

18. Verify that the function $u(x, y) = x^2 - y^2$ is harmonic. Also find v, the harmonic conjugate of u.

19. Using ML-inequality find an upper bound for the absolute value of $\oint_C \frac{e^z}{z^2+1} dz$, where C is the circle $|z| = 5$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. The position of a moving particle is given by $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 3t\mathbf{k}$. Find the vectors $\mathbf{T}, \mathbf{N}$ and $\mathbf{B}$. Also find the curvature.

21. State Cauchy's integral formula. Using Cauchy's integral formula evaluate,

    a. $\oint_C \frac{1+2e^z}{z} dz$ where C is $|z| = 1$.

    b. $\oint_C \frac{e^z}{z - \pi i} dz$ where C is $|z| = 4$.

(1 × 10 = 10 Marks)

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