22U506 (Pages: 2) Name: ……..…………..……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20U MTS5 B06 – BASIC ANALYSIS

(Mathematics – Core Course)

(2020 Admission onwards)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Let $S = \{1, 2\}$ and $T = \{a, b, c\}$. Determine the number of different injections from S into T.
2. Determine the set A of $x \in R$ such that $|2x + 3| < 7$.
3. Find $\inf S$ and $\sup S$ if $S = \{1 - (-1)^n; n \in N\}$.
4. Write the completeness property of R.
5. If x and y are real numbers with $x < y$, then prove that there exists an irrational number z such that x < z < y.
6. State the nested intervals property.
7. Show that $\lim\left(\frac{1}{n}\right) = 0$.
8. Evaluate $\lim(\frac{2n+1}{n})$.
9. Check whether the sequence $X = ((-1)^n)$ is convergent or not.
10. Define a Cauchy sequence.
11. The intersection of infinitely many open sets in R is open. Is it true or false? Justify your answer.
12. What is the reciprocal of $z = 2 - 3i$.
13. Describe the set of points z in the complex plane that satisfy the equation $|z - i| = |z - 1|$.
14. Express $-\sqrt{3} - i$ in polar form.
15. Evaluate the boundary points of $\{z \in C / Re\,(z) \geq 1\}$.

(Ceiling: 25 marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. Let the function f be defined by $f(x) = \frac{2x^2+3x+1}{2x-1}$ for $2 \leq x \leq 3$. Find a constant M such that $|f(x)| \leq M$ for all x satisfying $2 \leq x \leq 3$.
17. Prove that there does not exist a rational number r such that $r^2 = 2$.
18. State and prove Archimedean property.
19. Let $Y = (y_n)$ be defined inductively by $y_1 = 1$, $y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Show that $\lim Y = \frac{3}{2}$.
20. Show that every contractive sequence is a Cauchy sequence.
21. Prove that every Cauchy sequence converges in R.
22. Define principal argument of a complex number. Find $\arg(z)$ and $Arg(z)$.
23. Find the three cube roots of $z = i$.

(Ceiling: 35 marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. Suppose that S and T are sets and that $T \subseteq S$. If S is a finite set then show that T is also a finite set.
25. Prove that the set R of real numbers is not countable.
26. State and prove Bolzano-Weierstrass theorem
27. Find the image of the half plane $Re\ (z) \geq 2$ under the complex mapping $w = iz$ and represent the mapping graphically.

(2 × 10 = 20 Marks)

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