22U507 (Pages: 2) Name: .....

Reg.No: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC20U MTS5 B07 - NUMERICAL ANALYSIS

(Mathematics - Core Course)

(2020 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Suppose that $f \in C[a,b]$ with $f(a).f(b) < 0$. Prove that the bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with $|p_n - p| \leq \dfrac{b-1}{2^n}$ when $n \geq 1$
2. Use algebraic manipulations to show that the function $g(x) = \dfrac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$ has a fixed point at p precisely when $f(p) = 0$ where $f(x) = x^4 + 2x^2 - x - 3$
3. Explain the advantages of Newton-Raphson method.
4. Given $f(x) = \sqrt{x+1}$, $x_0 = 0$ and $x_1 = 0.6$. Construct interpolation polynomials of degree at most one to approximate $f(0.45)$ and find the absolute error.
5. Using the forward-difference formula approximate the derivative of $f(x) = x^2 \ln x - 1$ at $x_0 = 1$ by considering $h = 0.2$. Compute the actual error occurred in the approximation.
6. Given $f(x) = e^{2x}$. By taking $h = 0.1$ and using midpoint formula find $f''(2.0)$ correct to four decimal places.
7. What do you meant by degree of precision of a quadrature formula?
8. Write the open Newton-Cotes formula for $n = 3$. What is its error term?
9. Solve the initial value problem $y' = \frac{2}{t}y + t^2e^t,\ 1 \leq t \leq 2,\ y(1) = 0.$
10. What do you meant by local truncation error of a difference method?
11. Use midpoint method to approximate $y(1)$ given $y' = \sin t + e^{-t},\ y(0) = 0$
12. When an m-step multistep method is said to be explicit?

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Use secant method to find solution of $2x\cos 2x - (x-2)^2 = 0$ for $2 \leq x \leq 3$ accurate to within 10^{-4}.

14. Find an approximate root of $f(x) = \cos x - x = 0$ using the method of false position by taking $p_0 = 0.5$ and $p_1 = \frac{\pi}{4}$.

15. Using Newton's divided difference formula construct an interpolating polynomials of degree three for the data given in the table,

x	-0.1	0.0	0.2	0.3
$f(x)$	5.30	2.00	3.19	1.00

Add $f(0.35) = 0.97$ to the table and construct the interpolating polynomial of degree four.

16. Write a short note on round-off error instability in approximating derivative using three-point midpoint formula.

17. Compare the Trapezoidal rule and Simpson's rule approximations to $\int_0^2 (1+x)^{-1}\, dx$. Determine the actual error of approximation.

18. Use Euler's method to approximate the solution of the initial value problem $y' = \cos 2t + \sin 3t,\ 0 \le t \le 1,\ y(0) = 1$ with $h = 0.25$. Obtain the actual solution and compare the actual error at each step to the error bound.

19. Use modified Euler's method to approximate the solution of the initial value problem $y' = t^2 + y,\ 0 \le t \le 1,\ y(0) = 1$, with $h = 0.5$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. Each question carries 10 marks.

20. (a) Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

x	0	0.2	0.4	0.6	0.8
$f(x)$	1	1.2214	1.4918	1.8221	2.2255

(b) Use the Newton backward-difference formula to approximate $f(0.65)$

(c) Use Stirling's formula to approximate $f(0.43)$

21. Use Runge-Kutta method of order four to approximate the solution of the initial value problem $y' = -ty + 4t/y,\ 0 \le t \le 1,\ y(0) = 1$, with $h = 0.5$. Compare the results to the actual values given the actual solution is $y = \sqrt{4 - 3e^{-t^2}}$

(1 × 10 = 10 Marks)

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