22U509 (Pages: 2) Name: ……..……….……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20U MTS5 B09 – INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(Mathematics – Core Course)

(2020 Admission onwards)

Time: 2 Hours Maximum: 60 Marks

Credit: 3

PART A

Answer ***all*** questions. Each question carries 2 marks.

1. Find the equation of the parabola whose focus $(5,3)$ and the directrix is
$$3x - 4y + 1 = 0$$
2. Polar equation of a conic is $r = \frac{12}{3+3\sin\theta}$. Identify the conic and find its directrix.
3. Show that the line $lx + my + n = 0$ is normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$
4. Determine the inverse of the Euclidean transformation given by
$$t(x) = \begin{pmatrix} \frac{3}{5} & \frac{-4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}\boldsymbol{x} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$
5. Show that an affine transformation maps straight lines to straight lines.
6. By synthetic division, find the quotient and reminder in the division of $10x^3 - 2x^2 + 3x - 1$ by $2x - 3$
7. If a and b are different and the polynomial $f(x)$ is separately divisible by $x - a$ and $x - b$, show that $f(x)$ is divisible by $(x - a)(x - b)$
8. Write biquadratic equation with roots $i, -i, 1 + i$ and $1 - i$.
9. Given the roots of the equation $4x^3 - 24x^2 + 23x + 18 = 0$ are in arithmetic progression, solve the equation completely.
10. Solve the cubic equation $x^3 - 3x + 1 = 0$.
11. Separate the roots of the equation $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x - 1 = 0$ using Rolle's Theorem.
12. Show that the equation $12x^7 - x^4 + 10x^3 - 28 = 0$ has at least four imaginary roots.

(Ceiling: 20 Marks)

PART B

Answer ***all*** questions. Each question carries 5 marks.

13. Let P be an arbitrary point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and focus$F(ae,0)$. Let M be the midpoint of FP. Prove that M lies on an ellipse whose centre is midway between the origin and F.
14. Show that the set E(2) of all Euclidean transformations of R^2 forms a group under the composition of functions.
15. Determine the affine transformation which maps the point (0, 0), (1, 0) and (0, 1) in R^2 to the points (3,2), (5,8) and (7,3) respectively.
16. Factorize the polynomial x^6-1 into linear factors.
17. If $\alpha,\beta,\gamma,\delta$ are the roots of the equation $2x^4-6x^3+5x^2-7x+1=0$, evaluate $\alpha^2+\beta^2+\gamma^2+\delta^2$, $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ and $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}+\frac{1}{\delta^2}$
18. Find an upper limit of the positive real root of the equation
$$x^5-7x^4-100x^3-1000x^2+10x-50$$
19. Solve $x^4-2x^3-12x^2+10x+3=0$ by Ferrari's method.

(Ceiling: 30 Marks)

PART C

Answer any ***one*** questions. Each question carries 10 marks.

20. Classify the conic in R^2 with equation $x^2-4xy+4y^2-6x-8y+5=0$. Determine the centre or vertex and its axis.
21. Find the rational roots of the equation $25x^4-70x^3-126x^2+414x-243=0$.

(1 × 10 = 10 Marks)

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