24P153 (Pages: 2) Name: .....

Reg.No: .....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C01 / CC22P MST1 C01 - ANALYTICAL TOOLS FOR STATISTICS – I

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Prove that the function $f(x,y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at the origin.
2. State Taylor's theorem for multivariable functions and explain directional derivatives.
3. What do you mean by the method of Lagrangian multipliers? Explain.
4. What do you mean by Cauchy's integral formula? Explain.
5. Establish Morera's theorem.
6. What do you mean by singular point? Briefly explain different types of isolated singularities.
7. Find the residues of $f(z) = \frac{3z-1}{(z+1)^3(z-2)}$.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Find the complex function where the real part is given by $u_x = e^x(cosy - ysiny)$, given that the function is analytic.
9. Derive the polar form of Cauchy-Reimann equation.
10. Prove that $\int_0^\infty \frac{x\ sin\ ax}{x^2+k^2}dx = \frac{\pi}{2}\ e^{-ak}, (where\ a > 0, k > 0)$.
11. Evaluate a) $\int_0^\infty t^3 e^{-t} sint\ dt$ b) $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\ dt$
12. Find the inverse Laplace transform of $\frac{s+1}{s^2+6s+25}$ and $\frac{s+8}{s^2+4s+5}$.
13. Find the Fourier $sine$ transformation of the function $f(x) = \begin{cases} x & ,1 < x < 1 \\ 2-x & ,1 < x < 2 \\ 0 & ,x > 2 \end{cases}$.
14. Find the finite Fourier $sine$ and $cosine$ transform of $f(x) = 1,\ 0 < x < \pi$.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Establish Poisson's integral formula.

16. State and prove Taylor's theorem.

17. (a) State and prove the Cauchy Residue theorem.
    (b) State and prove Jordan's lemma.

18. Solve the initial value problem using the Laplace transform
    $(i)\ y'' + y' - 6y = 1, y(0) = 0, y'(0) = 1$
    $(ii)\ y'' + y = t, y(0) = a, y'(0) = 1, y(\pi) = 0$

(2 × 5 = 10 Weightage)

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