24P154 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C02 / CC22P MST1 C02 - ANALYTICAL TOOLS FOR STATISTICS – II

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Let V be a finite dimensional vectorspace and let $(v_1, v_2,...v_n)$ be any basis. Show that

   (i) If a set has more than n vectors; then it is linearly dependent.

   (ii) If a set has fewer than n vectors; it does not span V

2. Check whether the subsets S= $\{(x_1, x_2, x_3); 2x_1 +x_2 +x_3 = 1\}$ form subspace of vector space V = R^3.

3. Define trace of a square matrix. State any four properties relating to trace.

4. Define symmetric matrix. Give any six properties of symmetric matrix.

5. Prove that (i) Charecteristic root of idempotent matrix is either zero or one.

   (ii) If $\lambda \neq 0$ is a characteristic root of matrix A then $\frac{1}{\lambda}$ is a charecteristic root of A^{-1}.

6. State and prove rank-nullity theorem.

7. Illustrate different types of quadratic forms and write characteristics of each type of quadratic form.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Define norm. Prove for any inner product space, $||x|| = +\sqrt{< x, x >}$ is a norm.

9. Find inverse of matrix A. Given A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$.

10. Explain elementary operations of a matrix. Reduce the following matrix in to row reduced echelon form.

    $$A = \begin{bmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & 13 \end{bmatrix}$$

11. Define diagonalization of matrix. Prove that every non- zero nilpotent matrix is not diagonalizable.

12. Explain spectral decomposition of a matrix.

13. Define g- inverse. Show that $\bar{A}$ is the g-inverse of A if and only if $A\bar{A}A = A$.

14. Prove that If A is a real symmetric matrix of order m with rank r, then there exist a non singular matrix P such that $P^T AP$ = diag(1,1,1,....1,-1,-1,-1,.....,-1,0,0,0,,,,0).

(4 × 3 = 12 Weightage)

Part-C

Answer any **two** questions. Each question carries 5 weightage.

15. (a) State and prove rank nullity theorem.

(b) Reduce the matrix in to normal form and find rank. $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$.

16. (a) Define similar matrices. Prove similar matrices have the same minimal polynomial.

(b) Show that the eigen values of real symmetric matrix or a complex hermitian matrix are real.

(c) Show that any two characteristic vectors corresponding to two distinct characteristic roots of a real symmetric matrix are orthogonal.

17. (a) Define algebraic and geometric multiplicities of a characteristic root of an n x n matrix. Show that for any characteristic root geometric multiplicity cannot exceed the algebraic multiplicity.

(b) Let A be a square matrix of order n with distinct characteristic roots $\lambda_1, \lambda_2, .., \lambda_n$. Show that there exist an invertible matrix P such that $P^{-1}AP$= diag($\lambda_1, \lambda_2, .., \lambda_n$.)

18. (i) Define semi-definite quadratic form. Classify the quadratic form $5x^2 + 26y^2 + 10z^2 + 4yz + 14zx + 6xy.$

(ii) Define Gram matrix. Prove that every positive definite or positive semi definite matrix can be expressed as a gram matrix.

(2 × 5 = 10 Weightage)

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