24P155 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C03 / CC22P MST1 C03 - DISTRIBUTION THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Identify a discrete distribution for which mean= variance. State the reproductive property for this distribution.
2. Define multinomial distributions. Find its mean.
3. Define Weibull distribution. Obtain the distribution of U where $U = Min(X_1, X_2, \ldots, X_n)$ if $X_i^! s$ independently distributed according to standard Weibull.
4. Let X have a standard Cauchy distribution. Find the probability density function of X^2.
5. Let (X, Y) have the joint probability density function $f(x, y) = \frac{1}{4}$ for $|x|, |y| \leq 1$
   $= 0$ elsewhere

   Find marginal and conditional probability distributions.
6. Define mixture distributions. Discuss the practical situations where mixture distributions are appropriate.
7. Define sampling distribution and standard error. Obtain standard error of mean when population is large.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Define the probability generating function associated with a random variable. When will this reduce to i) characteristic function ii) moment generating function
9. Write down the probability mass function of the negative binomial distribution. Also obtain its mean and variance.
10. Write down the Beta probability functions of the first kind and second kind. Derive the r^{th} row moment of Beta distribution of II kind.
11. Write down the differential equation satisfied by the Pearson system of distributions.What is the basis for classification of member of the family into various type? Give an example.

12. Define bivariate normal distribution. If (X, Y) has a bivariate normal distribution, find the marginal density function of X.

13. A random sample of size n is taken from a population with distribution $f(x) = e^{-x}$, if $x \geq 0; = 0$ elsewhere. Find the probability density function of the range.

14. Let X and Y be independent χ^2 random variables with m and n d.f respectively. Find the distribution of $U = \frac{X}{Y}$

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. a) State and prove the reproductive property of Poisson distribution. Show that the mean and variance of the Poisson distribution are equal. Find the mode of the Poisson distribution with mean value 5.
    b) Show that for a Poisson distribution, the coefficient of variation is the reciprocal of the standard deviation.

16. Let X and Y denote a random sample of size 2 from $N(\mu, \sigma^2)$. Let U = X+Y and V= X-Y. Find the joint p.d.f of U and V and examine whether U and V are independent. Also evaluate the marginal distributions of U and V.

17. a) Obtain the characteristic function of the multivariate normal distribution and establish the reproductive property.
    b) If $X_1, X_2, \ldots, X_n$ are i.i.d random variables following $N(\mu, \sigma^2)$. Obtain the distribution of $\sum_{i=1}^{n} l_i X_i$ where $l_i's$ are constants. What can you say about independence of linear forms?

18. Derive moment generating function of non-central Chi-square distribution with 1 degrees of freedom.

(2 × 5 = 10 Weightage)

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