24P156 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MST1 C04 / CC22P MST1 C04 - PROBABILITY THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Given a class of 'n' sets $\{A_i, i = 1, 2, 3, \ldots n\}$. Show that there exists a class $\{B_i, i = 1, 2, 3, \ldots n\}$ of disjoint sets such that $\coprod_{i=1}^{n} A_i = \sum_{i=1}^{n} B_i$

2. Define independence of two events A and B. Prove that pairwise independence of events does not imply complete independence.

3. Test whether the following is a distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } 0 \leq x < \frac{1}{2} \\ 1, & \text{if } x \geq \frac{1}{2} \end{cases}$$

4. If X and Y are two independent simple random variables on same probability space $(\Omega, \mathscr{A}, P)$ then prove that *E(XY)= E(X).E(Y).*

5. Define convergence of a sequence of random variables $\{X_n, n \geq 1\}$ in law. Give an example to show that convergence in probability need not imply convergence in law

6. Distinguish weak convergence and complete convergence of a sequence of distribution functions $\{F_n, n \geq 1\}$. Show that the following seqeunce of distribution functions is weak convergent but not completely convergent $F_n(x) = \begin{cases} 0, \; if\; x < 0 \\ 1 - e^{\frac{-x}{n}}, \; if\; x \geq 0 \end{cases}$

7. Examine whether WLLN holds in the following sequence of independent random variables $\{X_n, n \geq 1\}$ where values and probabilities of X_n are given by $P(X_n = \pm n) = \frac{1}{2\sqrt{n}} \; and \; P(X_n = 0) = 1 - \frac{1}{\sqrt{n}}$

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. What do you mean induced probability space? Write down the induced probability space of a random variable X representing 'number of heads turned up' when a coin is tossed three times.

9. (a) State and prove Minkowski inequality.

   (b) State and prove Jenson's inequality.

10. Show that the distribution function of a random variable is symmetric if and only if its characteristic function is real. Verify this property in the case of standard normal distribution.

11. State and prove Borel Cantelli lemma.

12. Define convergence in probability. If $X_n \xrightarrow{P} X$ and $C \in \mathbb{R}$ is a constant, then show that $CX_n \xrightarrow{P} CX$.

13. State and prove Levy's continuity theorem for characteristic function.

14. State and prove Kolmogorov inequality.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Check whether $|\phi(t)|$ is integrable in the following case, and if so obtain the probability density function using inversion theorem $\phi(t) = e^{it-2t^2}$.

16. (a) Derive the characteristic function of negative binomial distribution.

    (b) Obtain the Taylor series expansion of characteristic function.

17. (a) If $X_n \xrightarrow{P} X$ then prove that $F_n(x) \xrightarrow{W} F(x), x \epsilon C(F)$.

    (b) If $X_n \xrightarrow{L} C$ where C is a constant, then show that $X_n \xrightarrow{P} C$.

18. State and prove Liapounov's form of central limit theorem.

(2 × 5 = 10 Weightage)

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