24P101 (Pages: 2) Name: ……………………

Reg. No: ……………………

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C01 – ALGEBRA – I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage

1. Check whether $\tau(x,y) = (x+3, y-5)$ is an isometry.
2. Find the order of the element $5 + \langle 4\rangle$ in $Z_{12}/\langle 4\rangle$.
3. State Burnside's Formula.
4. Show that the series $\{0\} < \langle 5\rangle < Z_{15}$ and $\{0\} < \langle 3\rangle < Z_{15}$ are isomorphic.
5. Prove that every group of order 15 is cyclic.
6. Find the reduced form and the inverse of the reduced form of the word $a^3b^{-1}b^5a^2c^3c^{-4}b^2$.
7. Find the units in $Z[x]$.
8. Give an example to show that the factor ring of an integral domain may be a field.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

Unit 1

9. If m divides the order of a finite abelian group G, then prove that G has a subgroup of order m.
10. Prove that M is a maximal normal subgroup of a group G if and only if G/M is simple.
11. Let X be a G-set. Prove that the relation $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$ is an equivalence relation on X.

Unit 2

12. State and prove the Third Isomorphism Theorem.
13. Let H be a p-subgroup of a finite group G. Prove that $(N[H]:H) \equiv (G:H)(mod\ p)$.
14. Prove that every group G' is the homomorphic image of a free group G.

Unit 3

15. Find all zeros of the polynomial $x^3 + 2x + 2$ in Z_7.

16. Prove that the multiplicative group of all nonzero elements of a finite field is cyclic.

17. Prove that the quaternions form a strictly skew field under addition and multiplication.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Let H be a subgroup of a group G.Prove that the left coset multiplication is well defined by the equation $(aH)(bH) = (abH)$ if and only if H is a normal subgroup of the group G.

    b) Prove that any subgroup H of G containing half the number of elements as that of G is a normal subgroup of that group G.

19. a) State and prove the Third Sylow Theorem.

    b) Let p and q be distinct primes. Prove that any group of order pq is not simple.

20. Determine all groups of order 10 up to isomorphism.

21. a) State and prove the Division Algorithm in $F[x]$.

    b) Show that the polynomial $x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over Q for any prime p

(2 × 5 = 10 Weightage)

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