24P102 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C02 - LINEAR ALGEBRA

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Let V be a vector space over the field F and α be any vector in V then prove that $(-1)\alpha = -\alpha$.
2. Describe the subspaces of $\mathbb{R}^3$.
3. Let V be vector space over the field and let T be a linear transformation from Vinto V. Prove that the inverse function T^{-1} is a linear transformation from V onto V.
4. Let T be a linear operator on $\mathbb{R}^2$ defined by$T(x_1, x_2) = (2x_1 + x_2, x_1 - 3x_2)$. Find the matrix of T in the standard ordered basis for $\mathbb{R}^2$.
5. Define transpose of a linear transformation. Prove that transpose of a linear transformation is again a linear transformation.
6. Define T conductor of α into W. Prove that $S(\alpha, W)$ is an ideal in the polynomial algebra$F[x]$.
7. Let $(|)$ be the standard inner product on $\mathbb{R}^2$. Let $\alpha = (1, 2), \beta = (-1, 1)$. If γ is a vector such that $(\alpha|\gamma) = -1$ and $(\beta|\gamma) = 3$, find γ .
8. Prove that an orthogonal set of non-zero vectors is linearly independent.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. If V is a finite dimensional vector space, then show that any two bases of V have the same number of elements.
10. Let $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for $\mathbb{R}^3$ consisting of $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (1, 0, 0)$.What are the coordinates of the vector (a, b, c) in the ordered basis $\mathcal{B}$
11. Let V and W be vector spaces over the field F and let T be a linear transformation from Vin to W. Suppose that V is finite dimensional Then prove that $\text{rank}(T) + \text{nullity}(T) = \dim V$

UNIT - II

12. Find the dual basis of $\mathcal{B}$. where $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be a basis for C^3 defined by $\alpha_1 = (2, 0, -2), \alpha_2 = (1, 1, 1), \alpha_3 = (1, 1, 0)$.

13. Let V be a finite dimensional vector space over the field F. . For each vector α in V define $L_\alpha(f) = f(\alpha),\ f \in V^*$. Then prove that the mapping $\alpha \to L_\alpha$ is an isomorphism of Vonto V^{**}.

14. Let T be a linear operator on the finite dimensional space V. Let $c_1, c_2, \ldots c_k$ be the distict characteristic values of T and let W_i be the space of characteristic vectors associated with the characteristic value c_i. If $W = W_1 + W_2 + \ldots + W_k$, then prove that $\dim W = \dim W_1 + \dim W_2 + \cdots + \dim W_k$

UNIT - III

15. If $V = W_1 \oplus W_2 \oplus \cdots \oplus W_k$, then prove that there exist k linear operators $E_1, E_2, \ldots, E_k$ on V such that

    1. Each E_i is a projection; 2. $E_iE_j = 0$ if $i \neq j$
    3. $I = E_1 + E_2 + \cdots + E_k$ 4. The range of E_i is W_i

    Conversely, prove that if $E_1, E_2, \cdots E_k$ are$k$ linear operators on V which satisfy conditions (i), (ii), (iii) and if we let W_i be the range of E_i, then prove that $V = W_1 \oplus W_2 \oplus \cdots \oplus W_k$

16. If V is an inner product space then for any vectors $\alpha$and β in V prove that $|(\alpha|\beta)| \leq ||\alpha||||\beta||$

17. If $W$is a finite dimensional subspace of an inner product sapace V and $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is any orthonormal basis for W, then prove that the vector $\alpha = \sum_{k=1}^{m} \frac{(\beta|\alpha_k)}{||\alpha_k||^2}\alpha_k$ is the unique best approximation to β by vectors in W..

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Show that $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$ where $W_1$and W_2 are finite dimensional subspace of a vector space.

19. (a) Let T be a linear transformation from V in to W. where V and $W$are finite dimensional and $\dim V = \dim W$ then prove that T is non-singular if and only if T is onto

    (b) Show that every n dimensional vector space over the field F is isomorphic to the space F^n.

20. Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, prove that the minimal polynomial divides the characteristic polynomial for T.

21. (a) Prove that the mapping $\beta \to \beta - E\beta$ is the orthogonal projection of V on $W^\perp$. where V is an inner product space, W a finite dimensional subspace, and E the orthogonal projection of V on W.

    (b) State and Prove Bessel's Inequality.

(2 × 5 = 10 Weightage)

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