24P103 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C03 - REAL ANALYSIS - I

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Let X be a metric space and $E \subset X$. Prove that $E = \overline{E}$ if and only if E is closed.
2. Every bounded infinite subset of R^k has a limit point in R^k.
3. If f is a continuous mapping from a metric space X to a metric space Y, then prove that f^{-1} is a continuous mapping from Y to X.
4. Mean value theorem holds always in the case of vector-valued functions. True or false. Justify your answer.
5. If P^* is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.
6. If $f \in \mathscr{R}$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that $F' = f$, then prove that $\int_a^b f(x)\, dx = F(b) - F(a)$.
7. Define rectifiable curve with example.
8. Prove Cauchy criterion for uniform convergence of sequence of functions.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that A is countable.
10. Prove that a set E is open if and only if its compliment is closed.
11. If P be a nonempty perfect set in R^k. Then prove that P is uncountable.

UNIT - II

12. State and prove generalised mean value theorem.
13. State and Prove L'Hospital's rule.
14. If f is monotonic on $[a, b]$and α is continuous on $[a, b]$, then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

UNIT - III

15. Prove that the series $\sum_{n=1}^{\infty}(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval.

16. If $f_n \to f$ uniformly on a set E and if x be a limit point of E and suppose that $\lim_{t\to x} f_n(t) = A_n$, for $n = 1, 2, 3, \cdots$. Then prove that $\{A_n\}$ converges, and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

17. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. If f is a continuous mapping of a compact metric space X into a metric space Y, then prove that $f(X)$ is compact and f is uniformly continuous on X.

19. (a) Prove that continuous image of a connected set is connected.

    (b) Prove that monotonic functions have no discontinuities of the second kind.

20. (a) If $f_1, f_2 \in \mathscr{R}(\alpha)$ on $[a, b]$, then prove that $f_1 + f_2) \in \mathscr{R}(\alpha)$ on $[a, b]$ and $\int_a^b (f_1 + f_2)\ d\alpha = \int_a^b f_1\ d\alpha + \int_a^b f_2\ d\alpha.$

    (b) If $f \in \mathscr{R}(\alpha)$ on $[a, b]$ and c is any constant, then prove that $cf \in \mathscr{R}(\alpha)$ on $[a, b]$ and $\int_a^b cf\ d\alpha = c\int_a^b f\ d\alpha.$

21. Prove Stone-Weierstrass theorem.

(2 × 5 = 10 Weightage)

****