FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH1 C04 - DISCRETE MATHEMATICS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours   Maximum : 30 Weightage

Part A

Answer **all** questions. Each question carries 1 weightage.

1. Show that intersection of two chains is again a chain.

2. Does there exist a boolean algebra with exactly 10 elements ?

3. Find the CNF of $f(x_1, x_2, x_3) = x_1 x_2 + x_2' x_3$.

4. Define a r edge connected graph with an example.

5. Prove that a connected graph is a tree if and only if it has only one face.

6. If te grith k of a connected plae graph G is at least 3, then $m \leq \frac{k(n-2)}{(k-2)}$.

7. Construct a DFA with a set of all strings that starts with zero.

8. Find an NFA with four states and $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.

(8 × 1 = 8 Weightage)

Part B

Answer any **two** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Let Y be the set of subsets of N, which are either finite or cofinite. Prove that Y is a subalgebra of the powerset boolean algebra $P(N)$.

10. Let X be a boolean algebra then $(X, \leq)$ is a complmented distributive lattice Coversely if $(X, \leq)$ is a bounded complimented and distributive lattice then there exist a boolean algebra such that the partial ordering relation defined by this structure coincides wth $\leq$.

11. Check whether the boolean function is symmetric; $x_1 x_2' x_3 x_4' + x_2 x_3' x_4 x_1'$.

UNIT - II

12. If G is simple and $\delta \geq \frac{n-1}{2}$, then G is connected.

13. A connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

14. Prove that $K_{3,3}$ is nonplanar.

UNIT - III

15. Let $\Sigma = \{a, b\}$. Consider two languages on Σ. $L_1 = \{a, aa, aaa\}$ and $L_2 = \{b, bb, bba, aab\}$. Find $L_1 \cup L_2, L_1 \cap L_2, L_1 - L_2, L_1^-$.

16. Show that $|u^n| = n|u|$ for all strings u and for all n.

17. If $L = \{awa : w\epsilon\{a, b\}^*\}$, then show that L^2 is regular.

(6 × 2 = 12 Weightage)

Part C

Answer any *two* questions. Each question carries 5 weightage.

18. The set of all symmetric boolean functions of n variables is a subalgebra of the boolean algebra of all boolean functions of these variables. As a boolean algebra it is isomorphic to the power set boolean algebra of the set $\{0, 1, 2, \ldots, n\}$.

19. State and prove Whitney's theorem on 2- connected graphs.

20. For a connected graph G the following statement are equivalent
    (a) G is Eulerian
    (b) The degree of each vertex of G is an even positive integer.
    (c) G is an edge disjoint union of cycles.

21. Contruct a DFA equivalent to the given NFA,
    $\delta(q_0, 0) = q_0, \delta(q_0, 0) = q_1, \delta(q_0, 1) = q_1, \delta(q_1, 0) = q_2, \delta(q_1, 1) = q_2, \delta(q_2, 1) = q_2$. Where q_1 is the final state.

(2 × 5 = 10 Weightage)

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